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Low-Pass to Band-Reject (Notch) Filter Transformation

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IN THIS MINI TUTORIAL

A transformation algorithm is available for converting low-pass poles into equivalent band-reject poles. One in a series of mini tutorials describing discrete circuits for op amps.

INTRODUCTION

As with band-pass filters, band-reject filters can be classified as either wide band or narrow band, depending on the separation of the poles. To avoid confusion, it is useful to adopt a convention. If the filter is wideband, it refers to as a band-reject filter. A narrowband filter is referred to as a notch filter.

In some instances, such as the elimination of the power line frequency (hum) from low level sensor measurements, a notch filter for a specific frequency may be designed.

Just as the band-pass case is a direct transformation of the low-pass prototype, where dc is transformed to F_0, the notch filter can be first transformed to the high-pass case, and then dc, which is now a zero, is transformed to F_0.

One way to build a notch filter is to construct it as a band-pass filter whose output is subtracted from the input (1 – BP). Another way is with cascaded low-pass and high-pass sections, especially for the band-reject (wideband) case. In this case, the sections are in parallel, and the output is the difference.

A more general approach is to convert the poles directly. A notch transformation results in two pairs of complex poles and a pair of second-order imaginary zeros from each low-pass pole pair.

The value of Q_{BP} is determined by

$$Q_{BR} = \frac{F_0}{BW} \tag{1}$$

where *BW* is the bandwidth at some level, typically −3 dB.

A TRANSFORMATION ALGORITHM

Given the pole locations of the low-pass prototype

$$-\alpha \pm j\beta \tag{2}$$

and the values of F_0 and Q_{BR}, the following calculations result in two sets of values for Q and frequencies, F_H and F_L, which define a pair of notch filter sections.

$$C = \alpha^2 + \beta^2 \tag{3}$$

$$D = \frac{\alpha}{Q_{BR} C} \tag{4}$$

$$E = \frac{\beta}{Q_{BR} C} \tag{5}$$

$$F = E^2 - D^2 + 4 \tag{6}$$

$$G = \sqrt{\frac{F}{2} + \sqrt{\frac{F2}{4} + D^2 E^2}} \tag{7}$$

$$H = \frac{DE}{G} \tag{8}$$

$$K = \frac{1}{2}\sqrt{(D+H)^2 + \quad (E+G)^2} \tag{9}$$

$$Q = \frac{K}{D+H} \tag{10}$$

The pole frequencies are determined by

$$F_{BR1} = \frac{F_0}{K} \tag{11}$$

$$F_{BR2} = K\ F_0 \tag{12}$$

$$F_Z = F_0 \tag{13}$$

$$F_0 = \sqrt{F_{BR1} \times F_{BR2}} \tag{14}$$

where F_0 is the notch frequency and the geometric mean of F_{BR1} and F_{BR2}.

A simple real pole, α_0, transforms to a single section having a Q given by

$$Q = Q_{BP}\ \alpha_0 \tag{15}$$

with a frequency $F_{BR} = F_0$. There is also transmission zero at F_0.

Assuming that an attenuation of A dB is required over a bandwidth of B, then the required Q for a single frequency notch is determined by

$$Q = \frac{\omega_0}{B\sqrt{10^{0.1A} - 1}} \tag{16}$$

The prototype is transformed into a band-reject filter. For this, the equation string above is again used. Each pole of the prototype filter transforms into a pole pair. Therefore, the 3-pole prototype, when transformed, has 6 poles (3 pole pairs).

As is the case with the band-pass, part of the transformation process is to specify the 3 dB bandwidth of the resultant filter.

Again, in this case this bandwidth is set to 500 Hz. The pole locations for the LP prototype were taken from the design table (see MT-206).

POLE LOCATIONS

The pole locations for the low-pass prototype were taken from the design table (see MT-206).

Table 1.

Stage	α	β	F_0	α
1	0.2683	0.8753	1.0688	0.5861
2	0.5366		0.6265	

The first stage is the pole pair and the second stage is the single pole. Note the unfortunate convention of using α for two entirely separate parameters. The α and β on the left are the pole locations in the s-plane. These are the values used in the transformation algorithms. The α on the right is 1/Q, which is what the design equations for the physical filters want to see.

The results of the transformation yield results as shown in Table 2.

Table 2.

Stage	F_0	Q	F_{0Z}
1	763.7	6.54	1000
2	1309	6.54	1000
3	1000	1.07	1000

Note that there are three cases of notch filters required. There is a standard notch ($f_0 = f_z$, section 3) a low-pass notch ($F_0 < F_Z$, section 1) and a high-pass notch ($F_0 > F_Z$, section 2). Since there is a requirement for all three types of notches, the Bainter notch is used to build the filter.

Figure 1 is the schematic of the filter. The response of the filter is shown in Figure 2 and in detail in Figure 3. Again, note the symmetry around the center frequency as well as the fact that the frequencies have geometric symmetry.

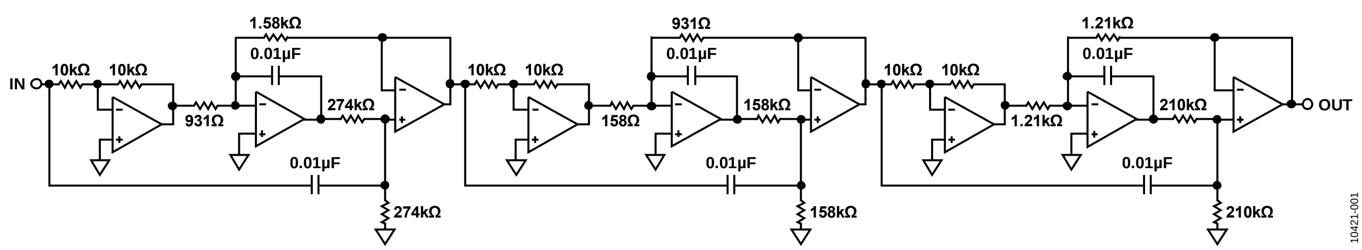

Figure 1. Band-Reject Transformation

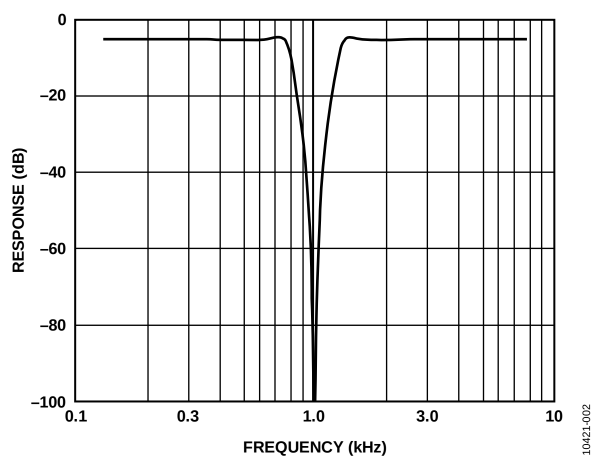

Figure 2. Band-Reject Response

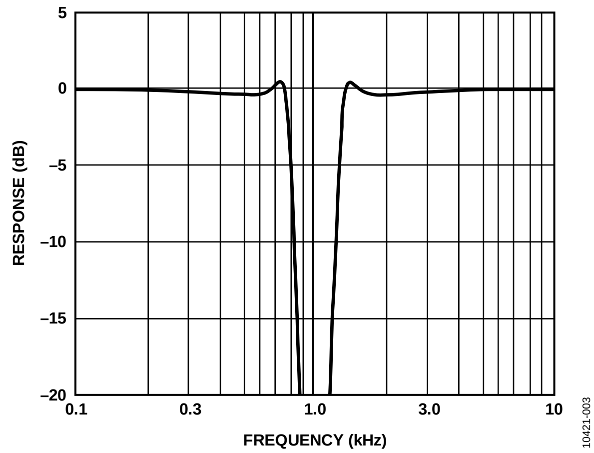

Figure 3. Band-Reject Response (Detail)

REVISION HISTORY

2/12—Revision 0: Initial Version