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The Butterworth Response

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IN THIS MINI TUTORIAL

The Butterworth filter, a precision op amp based circuit block, is one of multiple discrete circuits described in a series of mini tutorials.

INTRODUCTION

The Butterworth filter is the ideal compromise between attenuation and phase response. Because it has no ripple in the pass band or the stop band, it is sometimes referred to as a maximally flat filter. The Butterworth filter achieves its flatness at the expense of a relatively wide transition region from pass band to stop band, with average transient characteristics.

The normalized poles of the Butterworth filter fall on the unit circle (in the *s* plane). The pole positions are given by

$$-\sin\frac{(2K-1)\pi}{2n}+j\cos\frac{(2K-1)\pi}{2n}$$

where:
$K = 1, 2, \ldots n$ and is the pole pair number.
n is the number of poles.

The poles are spaced equidistant on the unit circle, which means the angles between the poles are equal. Figure 1 shows the pole placement for a 5-pole Butterworth filter.

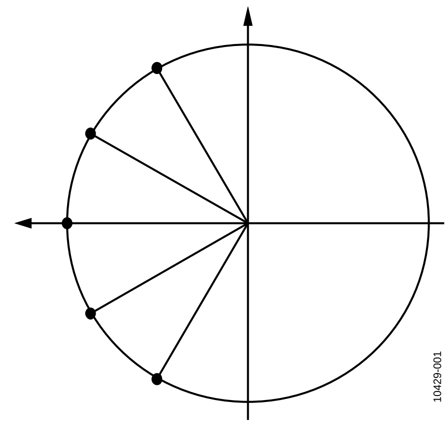

Figure 1. Butterworth Pole Location

Given the pole locations, ω_0 and α (or Q) can be determined. These values can then be used to determine the component values of the filter. The design tables for passive filters use frequency and impedance normalized filters. They are normalized to a frequency of 1 rad/sec and impedance of 1 Ω. These filters can be denormalized to determine actual component values. This allows the comparison of the frequency domain and/or time domain responses of the various filters on equal footing. The Butterworth filter is normalized for a −3 dB response at $\omega_0 = 1$.

The values of the elements of the Butterworth filter are more practical and less critical than many other filter types. The frequency response, group delay, impulse response, and step response are shown in Figure 2 through Figure 6. The pole locations and corresponding ω_o and α terms are tabulated in Table 1.

FREQUENCY RESPONSE, GROUP DELAY, IMPULSE RESPONSE, AND STEP RESPONSE

The frequency response, group delay, impulse response, and step response, as well as the amplitude, are cataloged in Figure 2 through Figure 6.

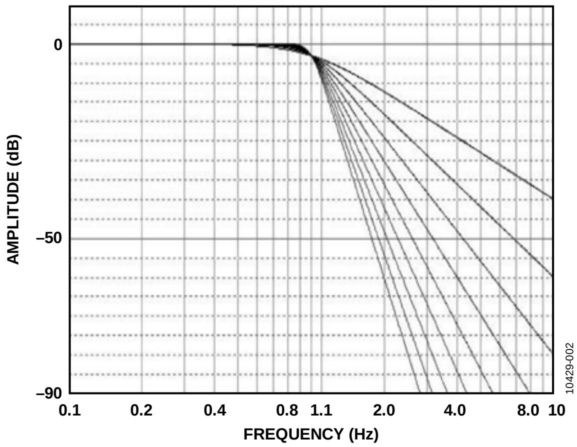

Figure 2. Butterworth Response, Amplitude

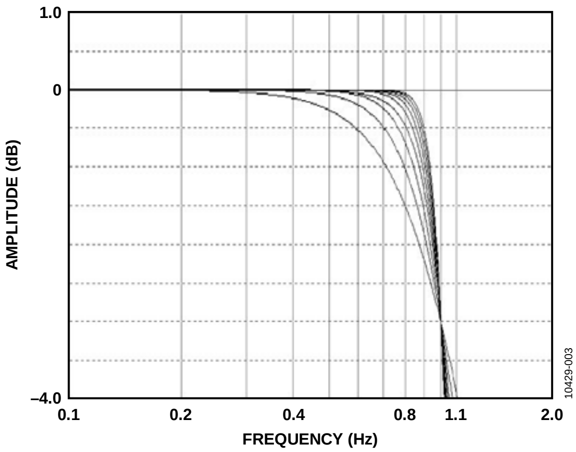

Figure 3. Butterworth Response, Amplitude Detail

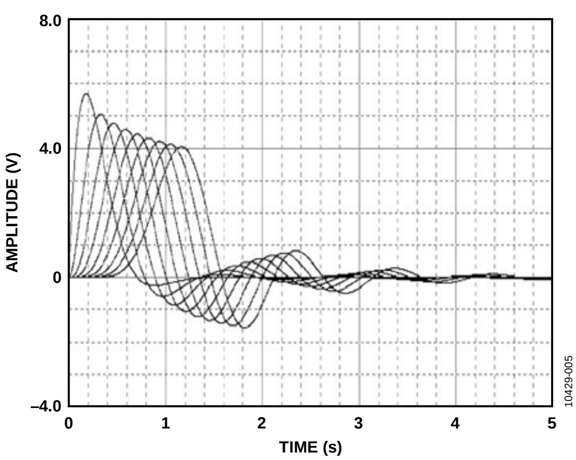

Figure 4. Butterworth Response, Impulse Response

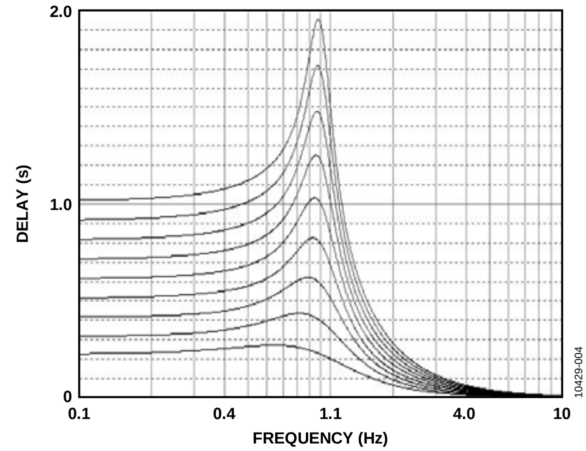

Figure 5. Butterworth Response, Group Delay

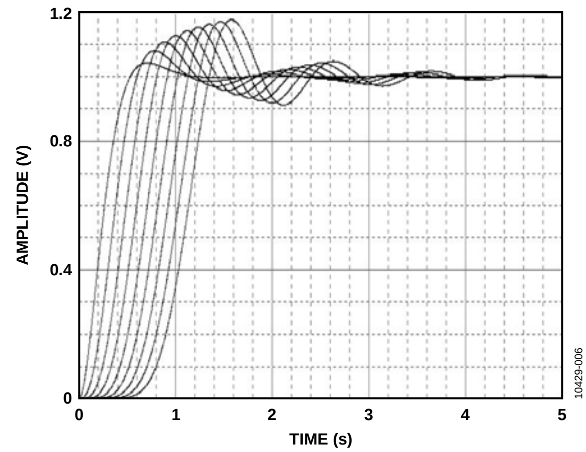

Figure 6. Butterworth Response, Step Response

BUTTERWORTH DESIGN

The pole location and corresponding ω and α terms for these values are tabulated in Table 1.

Table 1.

Order	Section	Real Part	Imaginary Part	F_0	α	Q	−3 dB Frequency	Peaking Frequency	Peaking Level
2	1	0.7071	0.7071	1.0000	1.4142	0.7071	1.0000		
3	1	0.5000	0.8660	1.0000	1.0000	1.0000		0.7071	1.2493
	2	1.0000		1.0000			1.0000		
4	1	0.9239	0.3827	1.0000	1.8478	0.5412	0.7195		
	2	0.3827	0.9239	1.0000	0.7654	1.3065		0.8409	3.0102
5	1	0.8090	0.5878	1.0000	1.6180	0.6180	0.8588		
	2	0.3090	0.9511	1.0000	0.6180	1.6182		0.8995	4.6163
	3	1.0000		1.0000			1.0000		
6	1	0.9659	0.2588	1.0000	1.9319	0.5176	0.6758		
	2	0.7071	0.7071	1.0000	1.4142	0.7071	1.0000		
	3	0.2588	0.9659	1.0000	0.5176	1.9319		0.9306	6.0210
7	1	0.9010	0.4339	1.0000	1.8019	0.5550	0.7449		
	2	0.6235	0.7818	1.0000	1.2470	0.8019		0.4717	0.2204
	3	0.2225	0.9749	1.0000	0.4450	2.2471		0.9492	7.2530
	4	1.0000		1.0000			1.0000		
8	1	0.9808	0.1951	1.0000	1.9616	0.5098	0.6615		
	2	0.8315	0.5556	1.0000	1.6629	0.6013	0.8295		
	3	0.5556	0.8315	1.0000	1.1112	0.9000		0.6186	0.6876
	4	0.1951	0.9808	1.0000	0.3902	2.5628		0.9612	8.3429
9	1	0.9397	0.3420	1.0000	1.8794	0.5321	0.7026		
	2	0.7660	0.6428	1.0000	1.5320	0.6527	0.9172		
	3	0.5000	0.8660	1.0000	1.0000	1.0000		0.7071	1.2493
	4	0.1737	0.9848	1.0000	0.3474	2.8785		0.9694	9.3165
	5	1.0000		1.0000			1.0000		
10	1	0.9877	0.1564	1.0000	1.9754	0.5062	0.6549		
	2	0.8910	0.4540	1.0000	1.7820	0.5612	0.7564		
	3	0.7071	0.7071	1.0000	1.4142	0.7071	1.0000		
	4	0.4540	0.8910	1.0000	0.9080	1.1013		0.7667	1.8407
	5	0.1564	0.9877	1.0000	0.3128	3.1970		0.9752	10.2023

REFERENCES

Zumbahlen, Hank. *Linear Circuit Design Handbook*. Elsevier. 2008. ISBN: 978-7506-8703-4.

REVISION HISTORY

4/12—Revision 0: Initial Version