ANALOG DEVICES

Mini Tutorial
MT-217

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Low-Pass to High-Pass Filter Transformation

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IN THIS MINI TUTORIAL

A transformation algorithm is available for converting low-pass poles into high-pass poles. One in a series of mini tutorials describing discrete circuits for op amps.

INTRODUCTION

Filters are typically described using the low-pass prototype because the low-pass configuration is the standard. A high-pass filter can be considered to be a low-pass filter turned on its side.

Instead of a flat response at dc, there is a rising response of n × (20 dB/decade), due to the zeros at the origin, where n is the number of poles. At the corner frequency a response of n × (−20 dB/decade), due to the poles, is added to the above rising response. This results in a flat response beyond the corner frequency.

The low-pass prototype is converted to a high-pass filter by scaling by 1/s in the transfer function. In practice, this typically amounts to capacitors becoming inductors with a value 1/C, and inductors becoming capacitors with a value of 1/L for passive designs. For active designs, resistors become capacitors with a value of 1/R, and capacitors become resistors with a value of 1/C. This applies only to frequency setting resistor, not those only used to set gain (that is, not every resistor or capacitor in the circuit).

A TRANSFORMATION ALGORITHM

Another way to look at the transformation is to investigate the transformation in the s plane. The complex pole pairs of the low-pass prototype are made up of a real part, α, and an imaginary part, β. The normalized high-pass poles are the given by

$$\alpha_{HP} = \frac{\alpha}{\alpha^2 + \beta^2} \quad (1)$$

and

$$\beta_{HP} = \frac{\beta}{\alpha^2 + \beta^2} \quad (2)$$

A simple pole, α_0, is transformed to

$$\alpha_{\omega,HP} = \frac{1}{\alpha_0} \quad (3)$$

Low-pass zeros, $\omega_{Z,LP}$, are transformed by

$$\omega_{Z,HP} = \frac{1}{\omega_{Z,LP}} \quad (4)$$

In addition, a number of zeros equal to the number of poles are added at the origin.

After the normalized low-pass prototype poles and zeros are converted to high-pass, they are then denormalized in the same way as the low-pass, that is, by frequency and impedance.

As an example a 1 kHz, 3-pole, 0.5d B Chebyshev filter is transformed. A Chebyshev is chosen because it shows more clearly if the responses were not correct; a Butterworth would probably be too forgiving in this instance. A 3-pole filter is chosen so that a pole pair and a single pole are transformed.

POLE LOCATIONS

The pole locations for the low-pass prototype were taken from the design table (see MT-206).

Table 1.

Stage	α	β	F_0	α
1	0.2683	0.8753	1.0688	0.5861
2	0.5366		0.6265	

The first stage is the pole pair and the second stage is the single pole. Note the unfortunate convention of using α for two entirely separate parameters. The α and β on the left are the pole locations in the s-plane. These are the values used in the transformation algorithms. The α on the right is 1/Q, which is what the design equations for the physical filters want to see.

The results of the transformation yield results as shown in Table 2.

Table 2.

Stage	α	β	F_0	α
1	0.3201	1.0443	0.9356	0.5861
2	1.8636		1.596	

A word of caution is warranted here. Since one convention of describing a Chebyshev filter (the convention used here) is to quote the end of the error band instead of the 3 dB frequency, the F_0 must be divided (for high pass) by the ratio of ripple band to 3 dB bandwidth.

The Sallen-Key high-pass topology is used to build the filter (see MT-222). The schematic is shown in Figure 1 .

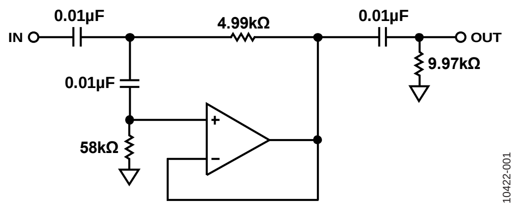

Figure 1. High-Pass Transformation

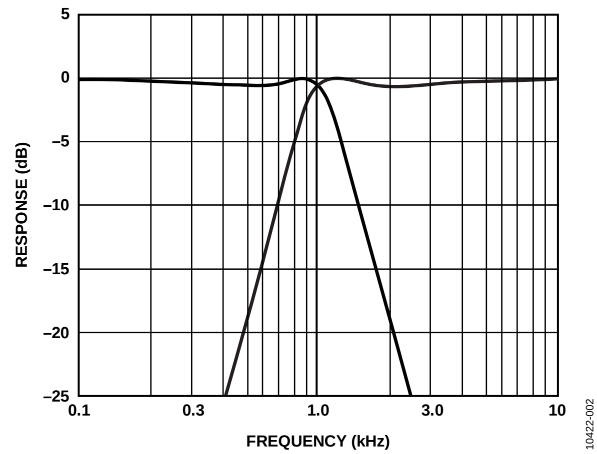

Figure 2. Low-Pass and High-Pass Response

Figure 2 shows the response of the low-pass prototype and the high-pass transformation. Note that they are symmetric around the cutoff frequency of 1 kHz. Also, note that the 0.5 dB error band is at 1 kHz, not the −3 dB point, a characteristic of Chebyshev filters. The symmetric nature of the responses verifies the accuracy of the transformation.

REVISION HISTORY

3/12—Revision 0: Initial Version