One Technology Way • P.O. Box 9106 • Norwood, MA 02062-9106, U.S.A. • Tel: 781.329.4700 • Fax: 781.461.3113 • www.analog.com

F_0 and Q in Filters

by Hank Zumbahlen,
Analog Devices, Inc.

IN THIS MINI TUTORIAL

The cutoff frequency (F_0) and quality factor (Q) of a filter are described in this mini tutorial, one in a series of mini tutorials documenting discrete circuits incorporating operational amplifiers (op amps).

The F_0 of a filter is defined as the cutoff frequency of the filter. Generally, this is the frequency where the amplitude response is down 3 dB from the pass band. For a Chebyshev filter, it can sometimes be defined as the frequency at which the amplitude response falls out of the pass band. For example, a 0.1 dB Chebyshev filter can have its F_0 defined at the frequency at which the response is down > 0.1 dB.

The shape of the attenuation curve (as well as the phase and delay curves, which define the time domain response of the filter) will be the same if the ratio of the actual frequency to the cutoff frequency is examined, rather than just the actual frequency itself. Normalizing the filter to 1 rad/s, a simple system for designing and comparing filters can be developed. The filter is then scaled by the cutoff frequency to determine the component values for the actual filter.

Q is defined as the quality factor of the filter. It is also sometimes given as α where:

$$\alpha = \frac{1}{Q} \tag{1}$$

This is commonly known as the damping ratio. Note that ξ is sometimes used where:

$$\xi = 2\alpha \tag{2}$$

If Q is > 0.707, there will be some peaking in the filter response. If the Q is < 0.707, roll off at F_0 will be greater; it will have a more gentle slope and the roll off will begin sooner. The amount of peaking for a 2-pole low-pass filter vs. Q is shown in Figure 1.

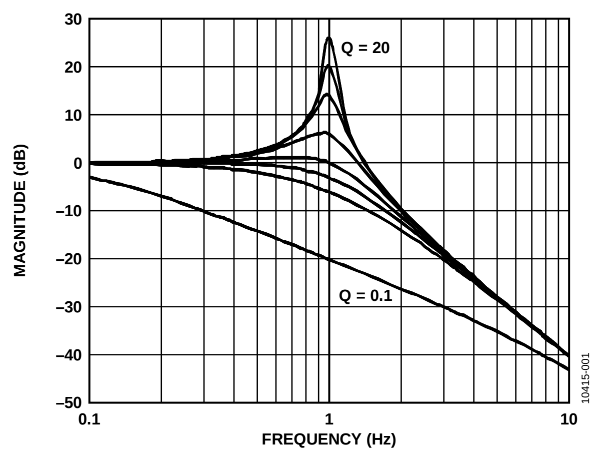

Figure 1. Low-Pass Filter Peaking vs. Q

Rewriting the transfer function H(s) in terms of ω_o and Q:

$$H(s) = \frac{H_0}{s^2 + \frac{\omega_0}{Q}s + \omega_0^{\,2}} \tag{3}$$

where H_0 is the pass-band gain and $\omega_o = 2\pi F_0$.

Now this the low-pass prototype will be used to design the filters.

HIGH-PASS FILTER

Changing the numerator of the transfer equation, H(s), of the low-pass prototype to H_0s^2 transforms the low-pass filter into a high-pass filter. The response of the high-pass filter is similar in shape to a lowpass, just inverted in frequency

The transfer function of a highpass filter is then

$$H(s) = \frac{H_0 s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^{\,2}} \tag{4}$$

The response of a 2-pole high-pass filter is illustrated in Figure 2.

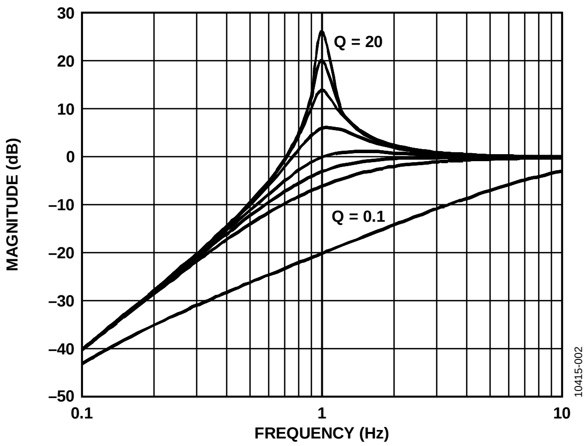

Figure 2. High-Pass Filter Peaking vs. Q

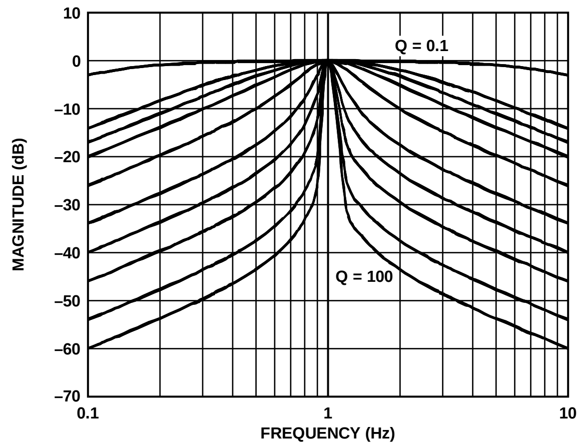

Figure 3. Band-Pass Filter Peaking vs. Q

BAND-PASS FILTER

Changing the numerator of the low-pass prototype to $H_o\omega_o^2$ converts the filter to a band-pass function.

The transfer function of a band-pass filter is then

$$H(s)=\frac{H_0\omega_0 s^2}{s^2+\frac{\omega_0}{Q}s+\omega_0{}^2} \tag{5}$$

where:

ω is the frequency ($F_0 = 2\ \pi\ \omega_0$) at which the gain of the filter peaks.

H_0 is the circuit gain and is defined

$$H_0 = H/Q. \tag{6}$$

Q has a particular meaning for the band-pass response. It is the selectivity of the filter. It is defined as:

$$Q=\frac{F_0}{F_H - F_L} \tag{7}$$

where F_L and F_H are the frequencies where the response is –3 dB from the maximum.

The bandwidth (BW) of the filter is described as

$$BW = F_H - F_L \tag{8}$$

Note that it can be shown that the resonant frequency (F_0) is the geometric mean of F_L and F_H, which means that F_0 will appear half way between F_L and F_H on a logarithmic scale.

$$F_0 = \sqrt{F_H\ F_L}$$

Also, note that the skirts of the band-pass response will always be symmetrical around F_0 on a logarithmic scale.

The response of a band-pass filter to various values of Q are shown in Figure 3.

A word of caution is appropriate here. Band-pass filters can be defined two different ways. The narrow-band case is the classic definition that as shown in Figure 3.

In some cases, however, if the high and low cutoff frequencies are widely separated, the band-pass filter is constructed out of separate high-pass and low-pass sections. Widely separated in this context means separated by at least two octaves (×4 in frequency). This is the wide-band case.

BAND-REJECT (NOTCH) FILTER

By changing the numerator to $s^2 + \omega_z^2$, one can convert the filter to a band-reject, or notch, filter. As in the band-pass case, if the corner frequencies of the band-reject filter are separated by more than an octave (the wide-band case), it can be built out of separate low-pass and high-pass sections. Thus, the following convention will be adopted: A narrow-band band-reject filter will be referred to as a notch filter and the wide-band band-reject filter will be referred to as band-reject filter.

A notch (or band-reject) transfer function is

$$H(s)=\frac{H_0(s^2+\omega_z{}^2)}{s^2+\frac{\omega_0}{Q}s+\omega_0{}^2} \tag{9}$$

There are three cases of the notch filter characteristics as illustrated in Figure 4. The relationship of the pole frequency, ω_0, and the zero frequency, ω_z, determines if the filter is a standard notch, a low-pass notch, or a high-pass notch.

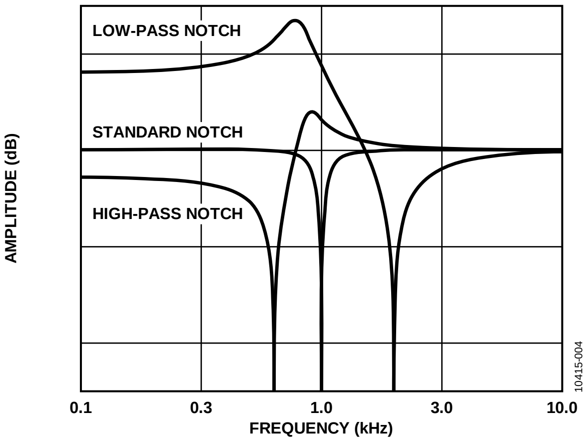

Figure 4. Standard, Low-Pass, and High-Pass Notches

If the zero frequency is equal to the pole frequency, a standard notch exists. In this instance, the zero lies on the *j*ω plane where the curve that defines the pole frequency intersects the axis.

A low-pass notch occurs when the zero frequency is greater than the pole frequency. In this case, ω_z lies outside the curve of the pole frequencies. What this means in a practical sense is that the filter's response below ω_z will be greater than the response above ω_z. This results in an elliptical low-pass filter.

A high-pass notch filter occurs when the zero frequency is less than the pole frequency. In this case, ω_z lies inside the curve of the pole frequencies. What this means in a practical sense is that the filters response below ω_z will be less than the response above ω_z. This results in an elliptical high-pass filter.

The variation of the notch width with Q is shown in Figure 5.

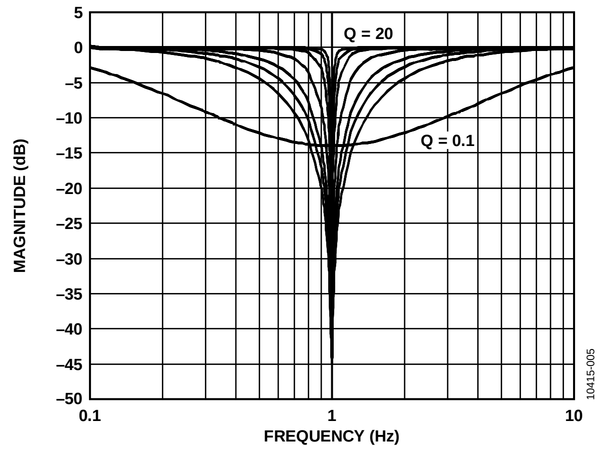

Figure 5. Notch Filter Width vs. Frequency for Various Q Value

REFERENCES

Zumbahlen, Hank. *Linear Circuit Design Handbook*. Elsevier. 2008. ISBN: 978-7506-8703-4.

REVISION HISTORY

1/12—Revision 0: Initial Version