# 3 Space Phasor Model Of PMSM

## 3.1. Introduction

For the purpose of understanding and designing vector controlled drives, it is necessary to know the dynamic model of the machine subjected to control. The machine models which are necessary to design control loops are very different from those used for designing the machine. Machine designers must have tolerance levels which are less than one percent, while control designs are only rough approximations, where even ten percent error can be considered acceptable. This is because every control scheme must absorb the changes of the plant parameters, due to changes in the temperature, supply, non-linearity etc. The effects of the load are only approximately considered.

However, a model of the electrical machine which is adequate for designing a control system must preferably incorporate all the important dynamic effects occurring during steady-state and transient operation. It should be valid for any arbitrary time variation of the voltages and currents generated by the converter which supplies the machine.

Such a model, valid for any instantaneous variation of voltage and current and adequately describing the performance of the machine under both steady-state and transient operation can be obtained by the utilization of space phasor theory.

**ASSUMPTIONS**

The following are assumed while modelling the PMSM:

1. There is no saturation
2. The induced EMF is sinusoidal
3. Eddy current and hysteresis losses are negligible
4. There is no cage or damper windings on the rotor.

## 3.2. Space Phasor Of Stator Currents

The three phase windings of the stator are represented diagrammatically in *figure* 3.1.

Each phase of the coil has a spatial orientation of $\gamma$, equal to 120 degree electrical with respect to the others.

The axis of phase-1 coil is taken as the reference for the spatial orientation. This forms the reference axis for a stationary co-ordinate system fixed to the stator. The currents in the windings can have any general variations with respect to time. Assuming that the spatial distribution of MMF produced by each coil is sinusoidal, the space phasor of the stator current in stationary coordinates is given by

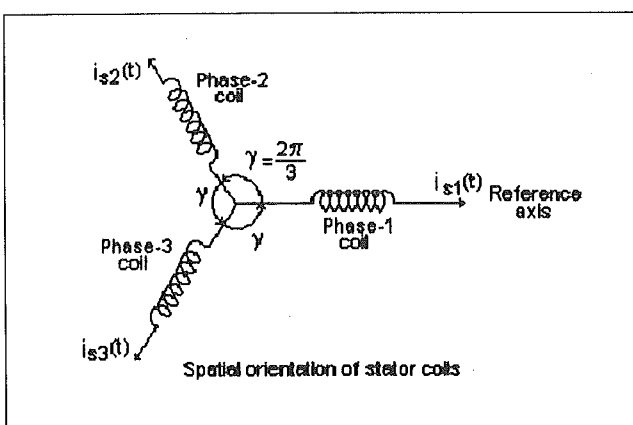



**Figure 3.1**

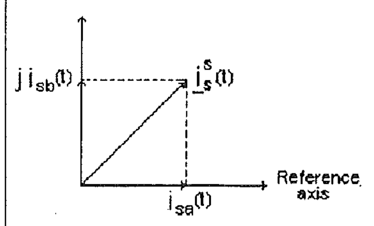
$$\underline{i}^s_s(t) = i_{s1}(t) + i_{s2}(t)e^{j\gamma} + i_{s3}(t)e^{j2\gamma} \quad ...3.2\text{-}1$$
$$= i_{sa}(t) + j\, i_{sb}(t) \quad ...3.2\text{-}2$$

where

$$i_{sa}(t) = i_{s1}(t) + i_{s2}(t) \cos \gamma + i_{s3}(t) \cos 2\gamma \quad ...3.2\text{-}3$$

$$i_{sb}(t) = i_{s2}(t) \sin \gamma + i_{s3}(t) \sin 2\gamma \quad ...3.2\text{-}4$$

From equation 3.2-2 it is seen that the space phasor of the stator currents is a complex function of time, whose real and imaginary parts give current along two mutually perpendicular directions in space as shown in *figure* 3.2.




Representation of space-phasor in a two dimensional plane

**Figure 3.2**

As $\gamma = 120°$, equations 3.2-3 and 3.2-4 can be rewritten as

$$i_{sa}(t) = i_{s1}(t) - (1/2)\, i_{s2}(t) - (1/2)\, i_{s3}(t) \quad ...3.2\text{-}5$$

$$i_{sb}(t) = (\sqrt{3}/2)\, i_{s2}(t) - (\sqrt{3}/2)\, i_{s3}(t) \quad ...3.2\text{-}6$$

For a three phase three wire system the condition

$$i_{s1}(t) + i_{s2}(t) + i_{s3}(t) = 0 \quad ...3.2\text{-}7$$

holds good for all instants of time. Using this condition in equations 3.2-5 and 3.2-6

$$i_{sa}(t) = (3/2)\, i_{s1}(t) \quad ...3.2\text{-}8$$

$$i_{sb}(t) = (\sqrt{3}/2)\, i_{s1}(t) + \sqrt{3}\, i_{s2}(t) \quad ...3.2\text{-}9$$

According to these equations, the space-phasor of the stator currents can be calculated from two of the three phase currents.

## 3.3. Space Phasor Of Rotor Currents

The rotor magnets can be considered as MMF sources; since they combine high electrical resistivity and unity permeability for external magnetic fields, they may be viewed as part of the airgap. Hence the motor is characterized by a constant wide airgap resulting in a relatively small synchronous reactance which minimizes armature reaction.

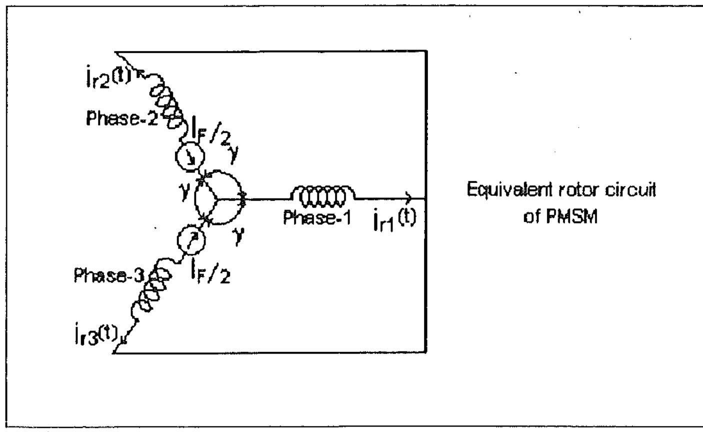



Figure 3.3

In order to obtain the space-phasor of the rotor currents, the MMF sources due to the magnets are considered to be equivalent current sources. Then, we can conveniently represent the permanent magnet rotor with three phase windings and two current sources, as shown in *figure* 3.3. The magnetic axis of the two pole rotor is taken as the reference for the spatial orientation of the fictitious rotor windings. This is also the reference axis (d-axis ) for a rotating co-ordinate system fixed to the rotor.

The space-phasor of the rotor currents in rotating co-ordinates is therefore given by

$$\underline{i}^r_r(t) = i_{r1}(t) + i_{r2}(t)e^{j\gamma} + i_{r3}(t)e^{j2\gamma} \qquad ...3.3\text{-}1$$

$$= I_F - (I_F/2)\, e^{j\gamma} - (I_F/2)\, e^{j2\gamma} \qquad ...3.3\text{-}2$$

As there are no MMF sources or magnets along the quadrature (q-axis), the imaginary part is zero and hence

$$\underline{i}^r_r(t) = (3/2)\, I_F \qquad ...3.3\text{-}3$$

The current $I_F$, is the equivalent current source representing the MMF of the magnet and is a function of the magnet dimensions and its magnetic properties.

## 3.4. Transformation Between Stator & Rotor Frames

The stator and the rotor current space-phasors are expressed with respect to two different co-ordinate systems, *viz.* the stationary co-ordinates fixed to the stator and the rotating co-ordinates fixed to the rotor respectively. The diagrammatic representation of the co-ordinate systems is shown in *figure* 3.4, for a two pole machine.

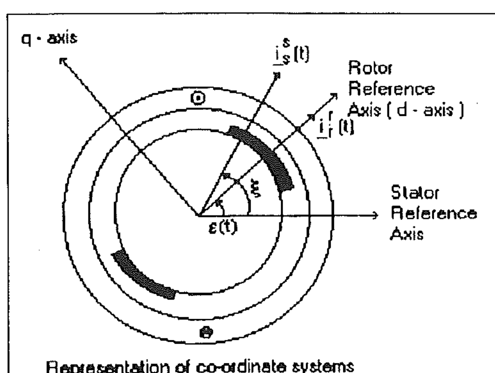



Figure 3.4

The instantaneous position of the rotor reference axis (d-axis ) with respect to the stator reference axis is $\epsilon$, say. The space-phasors defined in one co-ordinate system can always be transformed to another co-ordinate system. For the reference positions of the co-ordinate systems shown in *figure* 3.4 multiplication of rotor space-phasors by $e^{j\epsilon(t)}$ will transform them to the stationary co-ordinates. Conversely multiplication of the space phasors by $e^{-j\epsilon(t)}$ will transform them to the rotating co-ordinates.

## 3.5. Space Phasor Of Flux Linkages

The stator flux linkage space-phasor in stationary co-ordinates is given by

$$\underline{\Psi}^s_s(t) = L_s\, \underline{i}^s_s(t) + M\, [\, \underline{i}^r_r(t)\, e^{j\epsilon(t)}\, ] \qquad \text{...3.5-1}$$

where

$L_s$ - self (including leakage) inductance of the stator coils per phase

M - mutual inductance of the stator coils and the rotor.

As there are no windings on the rotor, M is the magnetizing inductance of the airgap including the magnet and is denoted as $L_o$ here.

Therefore,

$$\underline{\Psi}^s_s(t) = L_s\, \underline{i}^s_s(t) + L_o\, [\, \underline{i}^r_r(t)\, e^{j\epsilon(t)}\, ] \qquad \text{...3.5-2}$$

Substituting equation 3.3-3 in 3.5-2

$$\underline{\Psi}^s_s(t) = L_s\, \underline{i}^s_s(t) + L_o\, I_F\, e^{j\epsilon(t)} \qquad \text{...3.5-3}$$

$$= L_s\, \underline{i}^s_s(t) + \lambda_f\, e^{j\lambda(t)} \qquad \text{...3.5-4}$$

where $\lambda_f = (\,3/2\,)\, L_o\, I_F$ ...3.5-5

- the flux linkage due to rotor magnet flux linking the stator.

**SPACE-PHASOR OF STATOR VOLTAGES**

The stator voltage space-phasor in stationary co-ordinates is given by

$$\underline{u}^s_s(t) = R_s\, \underline{i}^s_s(t) + d/dt\, [\, \underline{\Psi}^s_s(t)\, ] \qquad \text{...3.5-6}$$

Substituting equation 3.5-4 in equation 3.5-6

$$\underline{u}^s_s(t) = R_s\, \underline{i}^s_s(t) + d/dt\, [\, L_s\, \underline{i}^s_s(t) + \lambda_f\, e^{j\epsilon(t)}] \qquad \text{...3.5-7}$$

$$\underline{u}^s_s(t) = R_s\, \underline{i}^s_s(t) + L_s\, d/dt\, [\, \underline{i}^s_s(t)\, ] + j\, \lambda_f\, d/dt\, [\, \epsilon(t)\, ] e^{j\epsilon(t)} \qquad \text{...3.5-8}$$

For a P-pole pair machine

$$d/dt\, [\, \epsilon(t)\, ] = P / 2 * \omega_r(t) \qquad \text{...3.5-9}$$

$$= \omega_s \qquad \text{...3.5-10}$$

where

$\omega_r$ - angular speed in mechanical rad/sec

$\omega_s$ - angular speed in electrical rad/sec

$\underline{u}^s_s(t) = R_s\,\underline{i}^s_s(t) + L_s\, d/dt\,[\,\underline{i}^s_s(t)\,] \;+ j\,(P/2)\,\lambda_f\,\omega_r(t)\,e^{j\varepsilon(t)}$ ...3.5-11

$= R_s\,\underline{i}^s_s(t) + L_s\, d/dt\,[\,\underline{i}^s_s(t)\,] \;+ j\,K_E\,\omega_r(t)\,e^{j\varepsilon(t)}$ ...3.5-12

where $K_E = (\,P/2\,)\,\lambda_f = (\,P/2\,)\,.\,(\,3/2\,)\,L_o\,I_F$ ...3.5-13

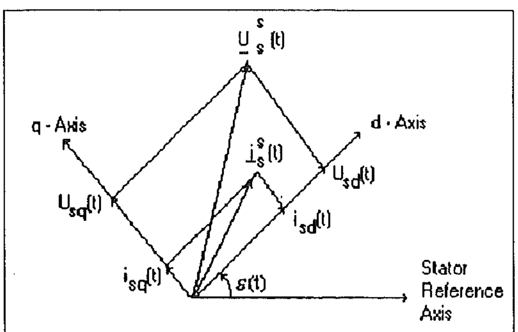
- The voltage constant expressed as (3/2) * Peak phase voltage per mechanical radians per sec.

The stator voltage and current space-phasors can be expressed in terms of the d and q axis components in the rotating co-ordinates as

$\underline{u}^s_s(t) = [\,u_{ds}(t) + ju_{qs}(t)\,]\,e^{j\varepsilon(t)}$ ...3.5-14

$\underline{i}^s_s(t) = [\,i_{ds}(t) + ji_{qs}(t)\,]\,e^{j\varepsilon(t)}$ ...3.5-15

where the subscripts d and q denote the quantities along the d and q axes of the rotating co-ordinates respectively as shown in *figure* 3.5.




Components of stator voltage and current space-phasors in rotating co-ordinates

**Figure 3.5**

Substituting equations 3.5-14 and 3.5-15 in equation 3.5-12

$[\,u_{ds}(t) + ju_{qs}(t)\,]\,e^{j\varepsilon(t)} = R_s\,[\,i_{ds}(t) + ji_{qs}(t)\,]\,e^{j\varepsilon(t)} + L_s\,d/dt\,\{[\,i_{ds}(t) + ji_{qs}(t)\,]\,e^{j\varepsilon(t)}\,\} \;+ j\,K_E\,\omega_r(t)\,e^{j\varepsilon(t)}$ ...3.5-16

$= R_s\,[\,i_{ds}(t) + ji_{qs}(t)\,]\,e^{j\varepsilon(t)}$

$+ L_s\,\{\,d/dt\,[\,i_{ds}(t) + j\,d/dt\,[\,i_{qs}(t)\,]\,\}\,e^{j\varepsilon(t)} \;+ j\,\omega_s\,L_s\,[\,i_{ds}(t) + i_{qs}(t)\,]\,e^{j\varepsilon(t)} \;+ j\,K_E\,\omega_r(t)\,e^{j\varepsilon(t)}$ ...3.5-17

where $\omega_s$ is given by equation 3.5-10.

Cancelling $e^{j\epsilon(t)}$ throughout, which amounts to transforming the equation to rotor coordinates, and separating real and imaginary parts,

$$u_{ds}(t) = R_s\, i_{ds}(t) + L_s\, d/dt\, [\, i_{ds}(t)\, ] - \omega_s\, L_s\, i_{qs}(t) \qquad \text{...3.5-18}$$

$$u_{qs}(t) = R_s\, i_{qs}(t) + L_s\, d/dt\, [\, i_{qs}(t)\, ] + \omega_s\, L_s\, i_{ds}(t) + K_E\, \omega_r(t) \qquad \text{...3.5-19}$$

Equations 3.5-18 and 3.5-19 represent the voltage equations of the machine in rotating co-ordinates.

## 3.6. Electromagnetic Torque

The electromagnetic torque developed by the machine is given by

$$T_e = (\,2/3\,)\,(\,P/2\,)\, L_o\, IM\, \{\, \underline{i}^s_s(t)\, [\underline{i}^r_r(t)\, e^{j\epsilon(t)}\,]^*\, \} \qquad \text{...3.6-1}$$

where

IM - represents the imaginary part

* - superscript represents the complex conjugate.

Substituting equation 3.3-3 in equation 3.6-1

$$T_e = (2/3)\,(P/2)\, L_o\, IM\, \{\, (3/2\;\; I_F\, [\underline{i}^s_s(t)\, e^{-j\epsilon(t)}]^*\, \} \qquad \text{...3.6-2}$$

where $\underline{i}^s_s(t)\, e^{-j\epsilon(t)}$ is the stator current space-phasor transformed to rotor co-ordinates and is given by

$$\underline{i}^s_s(t)\, e^{-j\epsilon(t)} = [\, i_{ds}(t) + j i_{qs}(t)\, ] \qquad \text{...3.6-3}$$

$$T_e = (P/2)\, L_o\, I_F\, i_{qs}(t)$$

$$= K_T\, i_{qs}(t)$$

where $K_T = (P/2)\, L_o\, I_F$

- torque constant of the machine expressed in Nm/Ampere and is given by rated torque / ( 3/2 * rated peak phase current ).

The torque equation is similar to that in a separately excited DC motor. Hence, the torque can be controlled independently by controlling the quadrature axis component of the stator current, maintaining the airgap flux at a constant value. In the case of DC machines and wound rotor synchronous motors field weakening is achieved by controlling the excitation current. But in the case of PMSM direct field weakening is not possible as the flux is derived from permanent magnets.

However a similar effect can be achieved by injecting a negative direct axis current. This is explained by considering the stator flux linkage equation 3.5-4.

Equation 3.5-4 can be rewritten in terms of the d and q axis components as

$$[\, \Psi_{sd}(t) + j\Psi_{sq}(t)\, ]\, e^{j\epsilon(t)} = L_s\, [\, i_{ds}(t) + j\, i_{qs}(t)\, ]\, e^{j\epsilon(t)} + \lambda_f\, e^{j\epsilon(t)} \qquad \text{...3.6-4}$$

Cancelling $e^{j\in(t)}$ throughout and separating the real and imaginary parts, the d and q axis flux linkages are given by

$$\Psi_{sd}(t) = L_s\, i_{ds}(t) + \lambda_f \qquad \text{...3.6-5}$$

$$\Psi_{sq}(t) = L_s\, i_{qs}(t) \qquad \text{...3.6-6}$$

Equation 3.6-5 implies that a negative direct axis current decreases the direct axis flux linkages, which amounts to field weakening.

The dynamic equation of the motor is given by

$$T_e = T_L + J\,(d\omega_r(t)/dt) + B\,\omega_r(t) \qquad \text{...3.6-7}$$

where
J - moment of inertia of the rotor in Kg $m^2$
B - friction and windage resistance in Nm/rad/sec.

Equations 3.5-18, 3.5-19, and 3.6-7 represent the dynamic mathematical model of the projecting type surface mounted PMSM.

## 3.7. Summary

The equations representing the PMSM in state-space form are as follows :

$$(di_{ds}(t)/dt) = (1/L_s)\,[\,U_{ds}(t) - R_s\, i_{ds}(t) + \omega_s\, L_s\, i_{qs}(t)\,]$$

$$(di_{qs}(t)/dt) = (1/L_s)\,[\,U_{qs}(t) - R_s\, i_{qs}(t) - \omega_s\, L_s\, i_{ds}(t) - K_E\,\omega_r(t)\,]$$

$$(d\omega_r(t)/dt) = (1/J)\,[\,T_e - T_L - B\,\omega_r(t)\,]$$

$$(d\in(t)/dt) = (P/2)\,\omega_r(t)$$

where $T_e = K_T\, i_{qs}(t)$

# NOTES