3 Induction Motors

3.1. Review Of Induction Motors

The squirrel cage induction motor is the industry work horse. Traditionally it has been a constant speed device. With the advent of static inverters, speed control through frequency variation has become practical, and affordable. This has increased the scope and range of applications. The basic theory of the squirrel cage induction motor is reviewed here.

3.2. Basic Principle of Operation

The induction motor has three phase windings on the stator and a rotor cage with end rings which electrically behaves as another three phase winding. When the stator windings are supplied with balanced three phase currents, a revolving magneto-motive force (*mmf*) is produced in the air gap. If the stator winding is designed properly, the spatial distribution of the stator *mmf* at any instant of time can be assumed to be sinusoidal. The location of the positive peak (north pole) of this *mmf* is taken as its instantaneous position. The speed of rotation of the stator *mmf* wave is given by

$N_e = 120f_s/P$ revolution per minute ...3.2-1

where: f_s is the frequency of the stator currents; P is the number of poles.

The stator *mmf* produces a rotating flux wave in the airgap of the machine. This flux wave, also, has a sinusoidal spatial distribution at any instant of time and rotates in space in the same direction and with the same speed as the stator *mmf*. The speed of the stator *mmf* (or the flux) is referred to as the synchronous speed.

As the flux sweeps past the rotor conductors, *emf*s are induced in these conductors. Since the rotor bars are shorted by the end rings, currents flow in the rotor conductors. The interaction of rotor currents and the flux produces torque and the rotor begins to turn. The rotor attempts to catch up with the flux. However, since this would result in the disappearance of torque, an equilibrium is reached where the rotor runs at a speed such that the relative motion of the flux is sufficient to produce enough torque to sustain the rotor speed. If the rotor runs at a speed of N revolutions per minute, the relative speed of the flux with respect to the rotor is given by

$N_r = N_s - N$ ...3.2-2

The slip 's' of the machine is defined as the ratio N_r/N_s.

i.e. $s = N_r/N_s = (N_s-N)/N_s = 1 - (N/N_s)$ ...3.2-3

The frequency of the currents in the rotor is given by

$f_r = (N_r/60)(P/2)$ ...3.2-4

Therefore, the slip can also be defined as

$$s = f_r/f_s \qquad ...3.2\text{-}5$$

ω_r and ω_s are the angular frequencies of the rotor and stator currents respectively. If ω is the speed of the rotor in electrical radians per second, slip is also defined as

$$s = \omega_r/\omega_s = (\omega_s - w)/\omega_s = 1 - \omega/\omega_s \qquad ...3.2\text{-}6$$

When currents are induced in the rotor, the rotor also produces an *mmf*. The net *mmf* acting on the air gap is the sum of the stator and rotor *mmfs*. The airgap flux is established due to the resultant *mmf*.

3.3. Mechanism Of Torque Production

When the motor is rotating in the steady state, the sinusoidally distributed airgap flux wave and the *emfs* induced in the rotor conductors can be indicated as shown in *figure* 3.1.

The airgap flux moves at a speed ω_r electrical radians/sec with respect to the rotor conductors. The amplitudes of the induced *emfs* in the rotor conductors are indicated in *figure*. Due to the induced voltage, current flows in the rotor conductors. However, because of the inductance in the rotor circuit, the rotor current is not in phase with the rotor induced *emf*, but lags it in time by an angle θ_r, which is the rotor power factor angle.

The inductance responsible for the presence of θ_r is the leakage inductance of the rotor. Consequently, the rotor conductor current amplitudes with respect to the airgap flux are as shown in *figure* 3.2.

The currents in two rotor conductors 180° apart can be regarded as current in a coil, producing an *mmf* along the axis of the coil. Therefore the rotor *mmf* wave will have a spatial position with respect to the airgap flux as shown in *figure* 3.3.

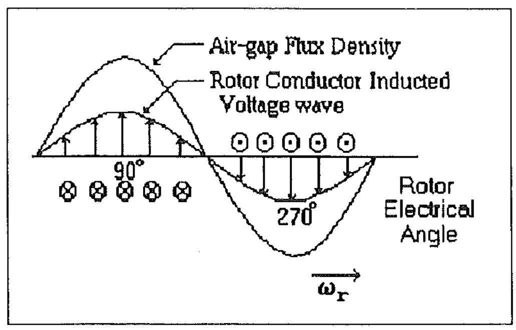

Figure 3.1

The rotor *mmf* also rotates at a speed ω_r with respect to the rotor, since the rotor currents alternate at the frequency ω_r. Therefore the rotor *mmf* will rotate at the synchronous speed ω_s with respect to the stator. The peak of the rotor *mmf* wave is at an angle $90+\theta_r$ behind the peak of the flux wave.

It is due to the lag between rotor voltage and rotor current, that the force on some of the rotor conductors opposes the rotation of the rotor.

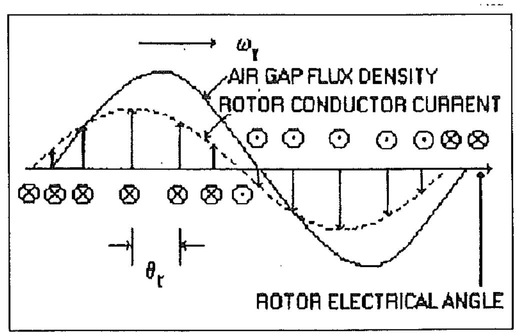

Figure 3.2

The net torque developed by the machine can be obtained by summing the forces acting on each of the rotor conductors. The expression for developed electromagnetic torque can be shown to be

$$m_d = \pi (P/2)\, lr\, B_p\, F_p \sin \delta \qquad ...3.3\text{-}1$$

where: P is the number of poles, l is the axial length of the machine, r is the radius of the rotor, B_p is the peak value of airgap flux density, F_p is the peak value of rotor *mmf* and $\delta = 90 + \theta_r$. ...3.3-2

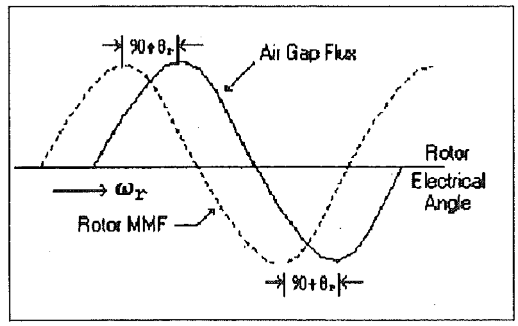

Figure 3.3

For the developed torque to be maximum for a given machine, the rotor power factor angle θ_r should be zero, but this is not possible in reality.
The torque equation can also be given as

$$M_d = 3/2(P/2)\, |\psi_m|\, |I_r| \sin \delta \qquad ...3.3\text{-}3$$

where $|\psi_m|$ is the peak value of airgap flux linkage per pole and $|I_r|$ is the peak value of rotor current.

To establish the airgap flux, stator *mmf* has to overcome the rotor *mmf* and produce a net magnetizing *mmf.*

In the case where the machine is operated from a voltage source, whenever rotor current flows, a corresponding current is drawn by the stator from the supply so as to maintain a net magnetizing *mmf.* Thus the machine acts like a transformer. Its behaviour can therefore be represented by an equivalent circuit similar to that of a transformer.

3.4. Steady State Equivalent Circuit

The airgap flux rotating at ω_s electrical radians per second induces a back *emf* V_m in the stator. The flux also moves at a speed ω_r with respect to the rotor. The corresponding *emf* induced in the rotor can be written as

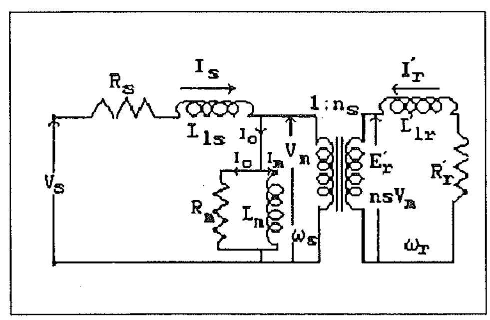

Figure 3.4

$$E'_r = n.s.V_m \qquad ...3.4\text{-}1$$

where n = rotor to stator turns ratio, s = slip

Therefore, the machine can be represented by an equivalent circuit as follows.

Here the two sides of the transformer work at different frequencies, namely, ω_s and ω_r.

The rotor current I'_r is given by

$I'_r = -(nsV_m) / (R'_r + j\omega_r L'_{lr})$ ...3.4-2

$I'_r = (nV_m) / ((R_r/s)+j(\omega_r/s)L'_{lr})$

$= (nV_m) / ((R'_r/s)+j\omega_s L'_{lr})$ ...3.4-3

Using eq. 3.4-2, the above equivalent circuit can be reduced to one in which both sides operate at the same frequency ω_s. In addition, the usual technique of referring all quantities to the primary (stator) turns gives the following equivalent circuit of the induction motor.

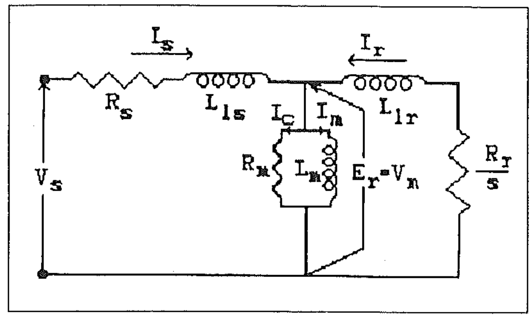

Figure 3.5

Note: *The rotor I_r is shown and not the component I_2 of stator current and $I_2 = -I_r$.*

Figure 3.6 shows a phasor diagram associated with the above equivalent circuit. For a machine with sinusoidally distributed windings, the time phase angles between different quantities in the phasor diagram translate into the spatial angles between them expressed in electrical radians. In the phasor diagram, the core loss component I_c of the stator current has been neglected for convenience.

The developed torque is now written as

$M_d = K.\ \psi_M\ I_r \sin\delta$
...3.4-4

where ψ_M and I_r are the rms values of airgap flux and referred rotor current respectively.

Therefore $M_d = K' I_M\ I_r \sin\delta$ ...3.4-5

Also, $I_r \sin\delta = I_s \sin\theta$ ...3.4-6

Thus, $M_d = K'\ I_M\ I_s \sin\theta$ ..3.4-7

$= K'\ I_M\ I_a$ ...3.4-8

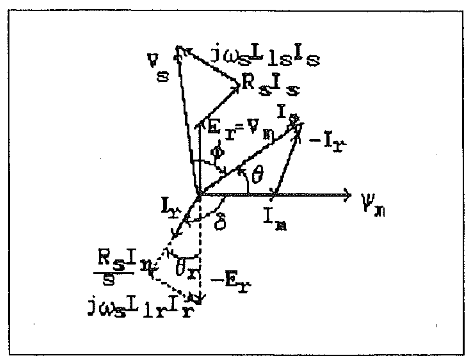

Figure 3.6

where I_a is the component of stator current in phase quadrature with the flux.

3.5. Steady State Equivalent Circuit Performance

The equivalent circuit of *figure* 3.5 can be approximated by neglecting the core-loss component and pushing the magnetizing inductance to the stator terminals. This circuit is more convenient to appreciate the performance of the machine. Based on this equivalent circuit, rotor current :

$$|I_r| = V_s/\{[R_s+(R_r/s)]^2 + \omega_s^2[L_{ls}+L_{lr}]^2\}^{1/2} \qquad ...3.5\text{-}1$$

The airgap power

$$P_{ag} = 3\,|I_r|^2\,R_r/s \qquad ...3.5\text{-}2$$

The electromagnetic torque

$M_d = P_{ag}$/synchronous speed

$$M_d = 3(P/2)\,(R_r/s\omega_s)\,(V_s^2/(R_s+(R_r/s))^2+\omega_s^2(L_{lr}+L_{ls})^2) \qquad ...3.5\text{-}3$$

The starting torque is obtained by setting s = 1 in the above equation. The torque speed characteristic based on eq. 3.5-2 is shown in *figure* 3.8.

Note : *For supersynchronous speeds of rotation (i.e. negative values of slip) the machine acts as a generator. The torque becomes negative, i.e., braking torque.*

In variable frequency drives, the machine is made to enter the generating region of the characteristics by reducing the stator frequency ω_s to a value below the running speed ω of the machine. In the plugging region, the field rotates in the opposite direction to the rotor; slip is greater than unity and there is excessive heating. The machine draws power from the electrical as well as the mechanical system to which it is connected.

The characteristics show that there is a value of slip s at which the torque is maximum, in the motoring mode as well as the generating mode. The maximum torque is called the pull-out torque and is usually about 2 to 3 times the rated torque of the machine.

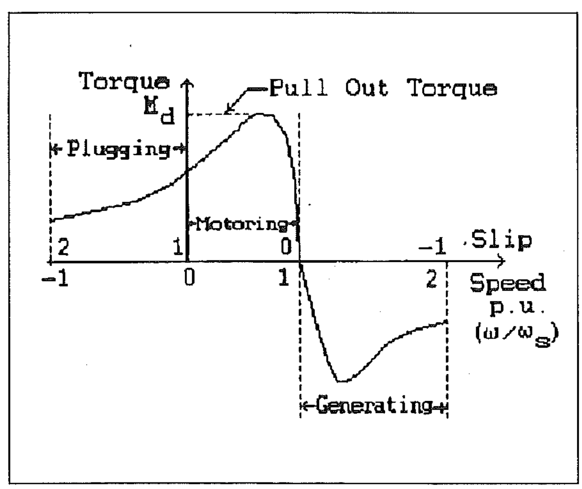

Figure 3.8

Further, approximate relationships can be obtained if the stator resistance and leakage inductance are neglected. Such an approximation is fairly accurate for integral horse power machines at speeds above 10% of rated speed. The approximate expression for torque is

$$M_d = 3(P/2)\,(V_s/\omega_s)^2\,\{(\omega_r.R_r)\,/\,([R_r^2 + (\omega_r L_{lr})^2]^{1/2})\} \qquad ...3.5\text{-}4$$

Rotor current

$I_r = (sV_s) / \{[R_r^2 + (\omega_r L_{lr})^2]^{1/2}\}$ ...3.5-5

$\cos \theta_r = R_r/[R_r^2 + (\omega_r L_{lr})^2]^{1/2}$ ...3.5-6

The airgap flux ψ_m is expressed as

$\psi_m = V_s/\omega_s$ ...3.5-7

For very small values of slip, $R_r >> \omega_r L_{lr}$ and the torque expression of eq. 3.5-3 simplifies to

$M_d = 3(P/2)\ \psi_m^2\ \omega_r\ (1/R_r)$ ...3.5-8

Therefore, at constant flux ψ_m, torque is directly proportional to the slip frequency ω_r. At constant ω_r, torque is proportional to the square of the flux.

3.6. Operating From Non Sinusoidal Sources

When an induction motor is operated from an inverter, the applied voltages are not sinusoidal. They contain odd harmonics, besides the fundamental. For example, in the six-step inverter, harmonics of order $6m \pm 1$ are present. Therefore, it becomes necessary to study the response of the machine to these harmonic voltages. The equivalent circuit offers a convenient starting point for this study. The equivalent circuit of the induction motor shown in *figure* 3.7 is redrawn in *figure* 3.9 for a general harmonic of order k.

All reactances get multiplied by the harmonic order k. It is assumed that resistance values remain the same, *i.e.* skin effect is neglected.

The value of the slip s_k has to be interpreted properly. The slip is the ratio of the rotor speed to the synchronous speed. The rotor speed is same, irrespective of the harmonic. But, the synchronous speed, *i.e.*, the speed at which the stator *mmf* is rotating, depends on the harmonic being considered. In general, the *mmf* due to kth harmonic rotates at k times the speed of the fundamental. However, not all the harmonic *mmfs* rotate in the same direction as that of the fundamental.

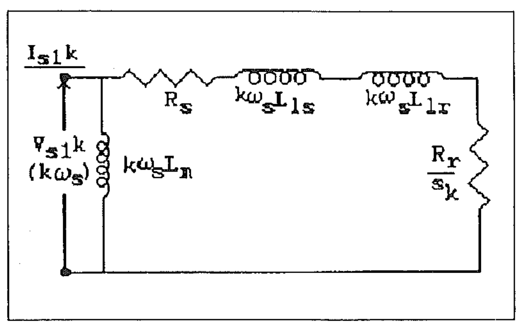

Figure 3.9

The harmonics of order 6m-1 (5,11,17, etc.) have a negative phase sequence. The *mmfs* produced by these harmonics rotate in the opposite direction to the fundamental. Therefore, the slip for the 5th harmonic is given by

$s_5 = (-5\omega_s - \omega) / (-5\omega_s) = 1 + (1/5)(\omega/\omega_s) = 1 + (1/5)(1-s_1)$ ...3.6-1

Since s_1 is very small, it can be approximated to

$s_5 = 1 + 1/5$

Similarly, $s_{11} = 1 + 1/11$; $s_{17} = 1 + 1/17$, etc.

For a positive sequence harmonic such as 7th,

$s_7 = (7\omega_s - \omega) / (7\omega_s) = 1 - (1/7)(\omega/\omega_s) = 1 - (1/7)(1-s_1)$ ...3.6-2

$s_7 = 1 - 1/7$

Similarly, $s_{13} = 1 - 1/13$; $s_{19} = 1 - 1/19$, etc.

In general, the slip corresponding to the harmonics is very close to unity. Therefore the equivalent circuit of *figure* 3.9 can be successively simplified. The resistances can be neglected in comparison with the leakage reactances at the harmonic frequencies. The magnetizing impedance can be neglected compared to the leakage reactances (in parallel connection). The machine offers only its leakage impedance to the harmonics in the applied voltage.

In a six-step inverter, the harmonics of the output voltage have amplitudes which are inversely proportional to the harmonic order. If X_1 is the leakage impedance of the machine at fundamental frequency, the various harmonic currents can be written as

$I_5 = (V_1) / (5.5X_1) = (V_1/X_1) / 25$

$I_7 = (V_1/X_1) / 49$; $I_{11} = (V_1/X_1) / 121$, etc. ...3.6-3

The total rms stator current is therefore given by

$I^2_{rms} = I_1^2 + (V_1/X_1)^2 [1/25^2 + 1/49^2 + 1/121^2 + ...]$

$= I_1^2[1 + (V_1/I_1X_1)^2 \{1/25^2 + 1/49^2 + 1/121^2 + ...\}]$ ...3.6-4

In a machine with 10% leakage impedance, V_1/X_1 corresponds to 10 times the rated current. The harmonic currents, expressed in p.u. with rated current as base, can be calculated as:

$I_5 = 10/25 = 0.4$ p.u. $I_7 = 10/49 = 0.2$ p.u. $I_1 = 10/121 = 0.083$ p.u. and so on.

The harmonic currents considerably increase the rms stator current and contribute to increased copper losses in the motor. Further, the maximum instantaneous current handled by the inverter switches can increase by a factor of 1.5 to 2 and the inverter devices have to be suitably rated. Some amount of derating for the whole inverter-drive system is inevitable.

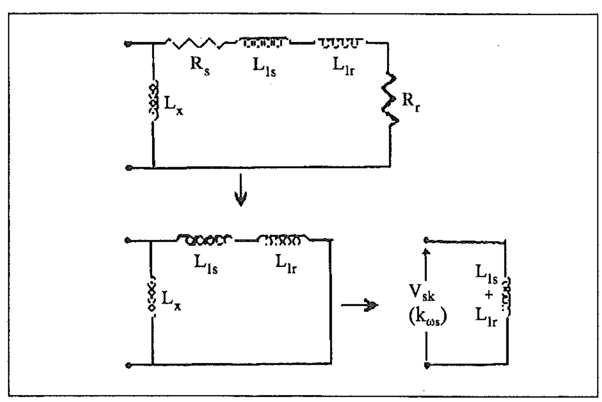

Figure 3.10

An important effect created by the current harmonics are the torque pulsations in the motor. The mechanism by which torque pulsations are produced is explained below.

3.7. Effect Of Harmonics on Torque

The basic mechanism of torque production in the motor is due to the interaction of rotor current with the airgap flux. In a machine excited by purely sinusoidal voltages, only fundamental flux and rotor current are produced and these rotate in synchronism, *i.e.*, a constant spatial angle is maintained between the flux and the current. Since torque is proportional to the magnitudes of the flux and the current and also the sine of the spatial angle between the two, a steady torque is produced.

In inverter fed machines, fluxes and currents of various harmonic frequencies are produced. A steady torque is produced by the interaction between flux at one frequency and current at the same frequency. In the case of positive sequence harmonics, this torque adds to the torque produced by the fundamental. In the case of negative sequence harmonics, it opposes the torque produced by the fundamental. However, the contributions of the harmonics to the steady output torque of the motor are negligible in magnitude. The useful output torque is only due to the fundamental.
In the interaction between flux at one frequency and rotor current at another frequency, two are in relative motion as they rotate at different speeds and possibly in opposite directions. The torque produced by such interactions pulsates with respect to time at the frequency of relative motion between the flux and torque considered. Phasor diagram of *figure* 3.11 shows such interactions.

The phasor diagram has the time origin at the instant where the fundamental flux ψ_{1m} has the peak value. Correspondingly, the fundamental applied voltage will be at its negative zero crossing and is therefore pointing upwards. The harmonic voltages V_5 and V_7 should also be at their negative zero crossings.

The voltage, V_5 is a negative sequence component and rotates in the clockwise direction and the phasor of V_5 points downwards. Once the voltage phasors are drawn, the corresponding flux phasors can be located at a lag angle of 90° with respect to the voltage and the direction of rotation.

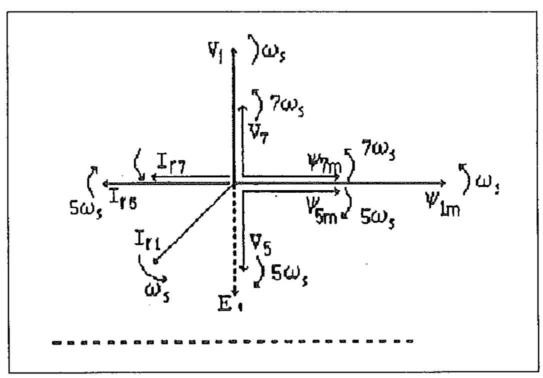

Figure 3.11

For each harmonic flux, the rotor induced *emf* lags by 90°. At the harmonic frequencies, the rotor leakage reactance dominates over rotor resistance and the rotor current lags the induced voltage by 90°. The currents I_{r5} and I_{r7} are depicted in *figure* 3.11 with these considerations.

The *figure* 3.11 can be given a clockwise rotation at a speed ω_s, thereby making the fundamental quantities stationary. The resulting diagram is shown in *figure* 3.12.

From *figure* 3.12, it is clear that I_{r5} and I_{r7} are rotating with respect to ψ_{1m} at 6 times the fundamental speed, but in opposite directions. The currents I_{r5} and I_{r7} produce torque components pulsating at 6 times the fundamental frequency ω_s. These torque components can be expressed as

$$M_{d6,1} = K[\psi_{1m} I_{r5} \sin(\pi + 6\omega_s t) + \psi_{1m} I_{r7} \sin(\pi - 6\omega_s t)] \qquad ...3.7\text{-}1$$

Similarly the flux components ψ_{5m} and ψ_{7m} will interact with I_{r1} and produce sixth harmonic torque pulsation. These components can be expressed as

$$M_{d6,2} = K[I_{r1}\ \psi_{5M} \sin(\delta - 6\omega_s t) + I_{r1}\ \psi_{7M} \sin(\delta + 6\omega_s t)] \qquad ...3.7\text{-}2$$

If δ is approximately taken as 90°, the total 6th harmonic torque pulsation due to all the four flux current

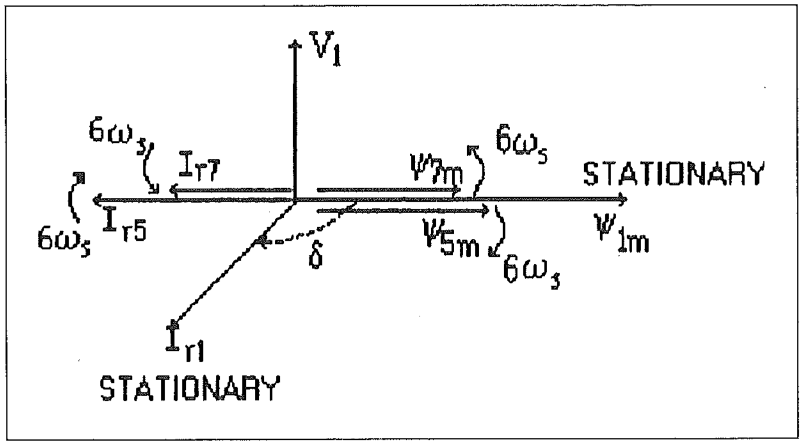

Figure 3.12

pairs is

$$M_{d6} = K[\psi_{1m}(I_{r7}-I_{r5}) \sin 6\omega_s t + I_{r1}(\psi_{7M}+ \psi_{5M})\cos 6\omega_s t] \qquad ...3.7\text{-}3$$

The harmonic fluxes ψ_{7M} and ψ_{5M} are generally very small and in any case the second term adds to the first in quadrature. The sixth harmonic torque pulsation can be expressed as

$$M_{d6} = K.\psi_{1M}(I_{r7} - I_{r5}) \sin 6\omega_s t \qquad ...3.7\text{-}4$$

The fundamental flux interacting with 7th and 5th harmonic fluxes, produces 6th harmonic torque pulsation. Note that there is a cancelling effect between the contribution of the two currents. Similarly torque pulsations at the 12th, 18th,... harmonics are also produced, although the 6th harmonic pulsation is the predominant one.

The time variation of the instantaneous developed torque in an induction machine fed by a six step inverter is shown in *figure* 3.13. It is even possible that the instantaneous torque is negative, for low values of average torque.

Such a pulsating torque may result in jerky motion of the motor, especially at low speeds (frequencies). Even at higher frequencies, torque pulsations are too fast for the mechanical system to respond and add to increased stress on the shaft and bearings.

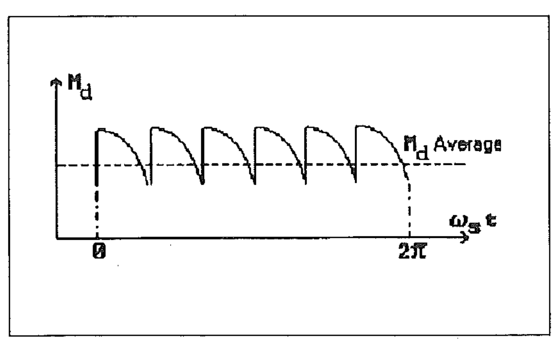

Figure 3.13

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