III.21 MONOSTABLE MULTIVIBRATOR. A monostable multivibrator has only one stable state. However, immediately following the introduction of a trigger pulse, it will change state and stay in the new "metastable" state for a predetermined period, and then return to its stable state.

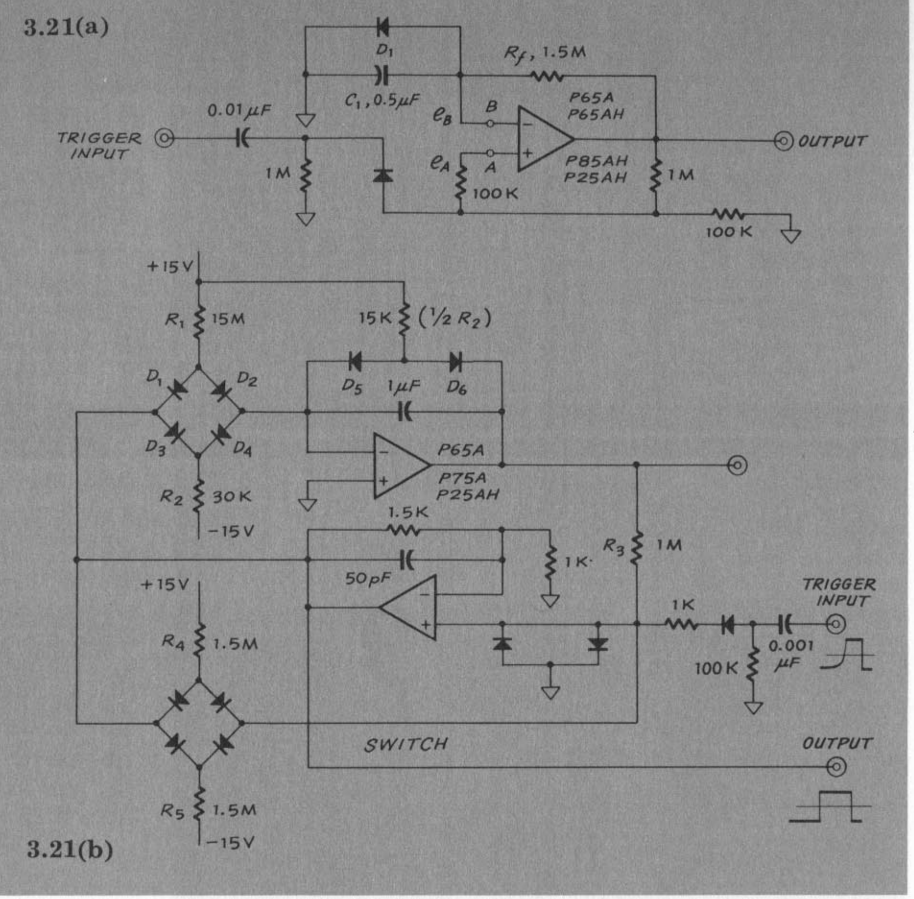
In circuit (a), the output states are plus and minus saturation (+12 V and −12 V, typically). When a negative step is applied to the trigger input (3 to 8 volts, typically) the output switches from its stable state at +12 V to −12 V, and R_f begins to charge C_1 toward −12 V. When the voltage on C_1 reaches a point at which $|e_B| > |e_A|$, the amplifier output voltage switches back to +12 V, and will remain there until the circuit is triggered again. Meanwhile, the voltage on C_1 starts to charge back up from −1 V to +12 volts, but is clamped at about +0.6 V, by D_1. Values of C_1 from 1000 pF to 10 μF and of R_f from 10 kΩ to 2 MΩ are suitable (for smaller values of R or C, a fast amplifier is recommended). With the values shown, the temporary state lasts about 1 second.

To obtain more predictable and "nearer-ideal" behavior, circuit 3.21 (b) is recommended. As in the nonsaturating flip-flop of 3.17(b), diode bounds are used to prevent saturation and improve response speed, trigger sensitivity, and stability. The circuit is a variation of the triangular-wave generator of III.15. In the stable state, the switch output is at about −1 VDC, and about 0.5 mA flows through D_4 and D_5. The integrator's output is bounded at about 0 V by D_5.

If a step signal of sufficient magnitude is applied to the trigger input, the output of the switch will go very quickly to +1 volt. Current then flows through R_1 and D_2, and the output of the integrator begins to ramp downward linearly. When it reaches −10 volts, the current through R_4 balances that through R_3, and the switch flips back to its permanent state of −1 V. This connects R_2, through D_4 resetting the integrator. Since R_2 can be much smaller than R_1, the reset can be accomplished quickly. Using the formula

$$\tau = 0.7 R_1 C \tag{3-6}$$

for the component values shown, the output will be at +1.0 volt for 10 seconds. Substituting the value of R_2 for R_1 in equation (3.6) gives the formula for the reset time—in this example, only 20 milliseconds.



3.21(a)

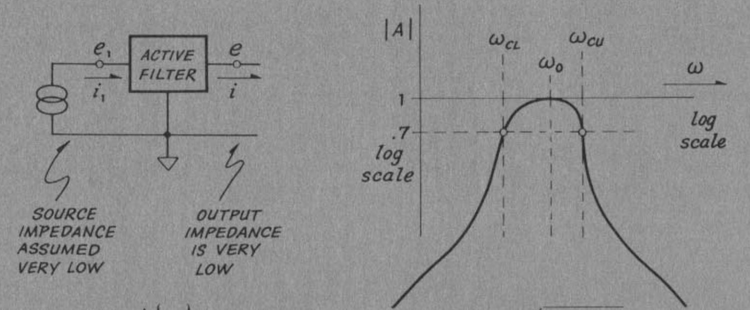
3.21(b)

III.21
III.17
III.18
III.20

III.22 ACTIVE FILTERS. Operational Amplifiers can greatly simplify the design of high-performance signal filters, because they eliminate the need for inductors and for impedance matching. Furthermore, use of active filters can result in reduction of weight, size, and cost. Filters designed to satisfy sophisticated mathematical criteria can be realized without resort to "equalization" or trimming.

We shall, in the six sections that follow, attempt to summarize a two-part article published in *The Lightning Empiricist*, Vol. 13, 1 & 2, 3 & 4. Our necessarily brief considerations will include Operational Amplifier circuits for low-pass, high-pass, band-pass, and band-reject filters. Procedures for cascading quadratic filter stages will be presented, so that high-order "mathematically-designed" filters, in this case the Butterworth, may be synthesized.

In section III.28, we present (in highly-concentrated form) filter-design tables and a design example, all from the second part of the article, in which all these matters are examined in a less compressed, more detailed manner.



$$e = A\{p\}e_1$$

$$pe_i \equiv \frac{de_i}{dt}$$

$$\omega_0 = \sqrt{\omega_{CU}\omega_{CL}}$$

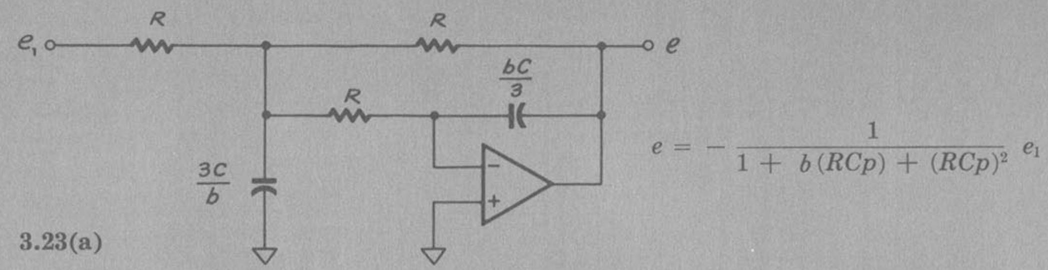
$$B = \frac{\omega_{CU} - \omega_{CL}}{2\omega_0}$$

3.22

III.22
III.23
to
III.28

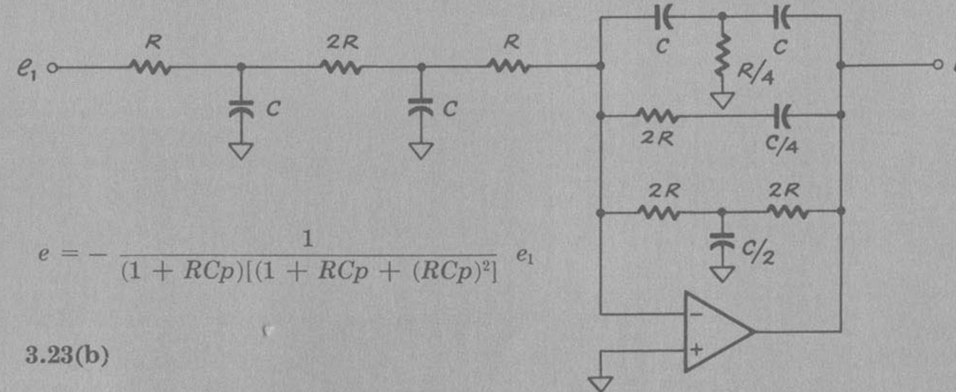
III.23 SIMPLE LOW-PASS FILTERS. The simplest of low-pass filters are the first-order lag circuits of II.15 and II.16. *Efficient* higher-order filters exhibit a response equation having no more than one real root, the other factors being damped quadratics (b < 2). Figure (a) shows a typical quadratic or second-order underdamped filter section. Several such sections may be cascaded to achieve a higher even-order (e.g., 4th, 6th, etc.) filter characteristic.

An example of a third-order filter circuit was given in II.18. Another is shown in (b). This circuit is a third-order Butterworth filter having relatively sharp cut-off and a flat band-pass region. Its response is 3 db down at $\omega = \dfrac{1}{RC}$ rad/sec. Another useful third-order filter is shown in III.69.



$$e = -\frac{1}{1 + b(RCp) + (RCp)^2}\, e_1$$

3.23(a)



$$e = -\frac{1}{(1 + RCp)[(1 + RCp + (RCp)^2]}\, e_1$$

3.23(b)

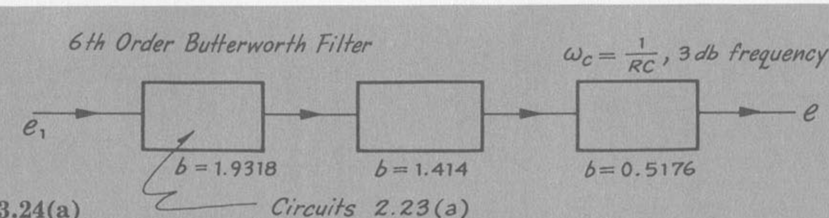
III.23
II.1
to
II.4
II.15
to
II.18
III.22
III.24
to
III.28

III.24 CASCADING ACTIVE FILTER SECTIONS. Cascading quadratic factors as indicated in (a) is an efficient way of achieving a filter with very high-performance characteristics, particularly when dealing with low-pass and band-pass designs. There is no need for the sections to be identical; in fact, Butterworth low-pass factors all have unique damping ratios $\left(\dfrac{b}{2}\right)$ and narrow-band-pass factors all have unique natural frequencies and damping ratios.

Values of b, the damping ratio, are tabulated in Table (b) for various even-order low-pass Butterworth filters. The absolute value of the gain, $|A|$, for these filters is given (as a function of frequency, ω) by:

$$|A|^2 = \frac{1}{1 + (\omega RC)^{2n}} \qquad (3\text{-}7)$$

The transient response of the Butterworth low-pass filter deteriorates in fidelity as the number of sections increases.



3.24(a)

Low-Pass Butterworth Factors

Order, n	b
2	1.414
4	1.845, 0.7654
6	1.932, 1.414, 0.5176
8	1.962, 1.663, 1.111, 0.3896
10	1.976, 1.783, 1.414, 0.9081, 0.3128

3.24(b)

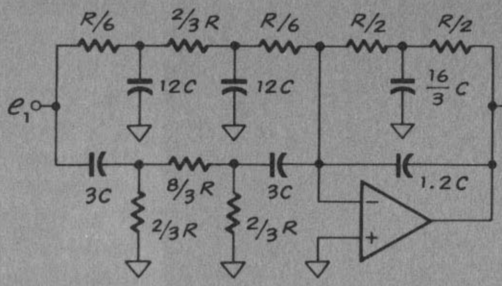
III.24
II.1
to
II.4
II.15
to
II.18
III.22
III.23
III.25
III.28

III.25 "AVERAGING" FILTERS. Circuit (a) is a modification of the third-order, linear-phase-shift (Paynter) filter shown in II.18. This has a notch at $\omega = \frac{1}{RC}$, achieving the amplitude vs. frequency characteristic shown in the heavy line in (b). The dotted line is the "ideal running-average" operator given by:

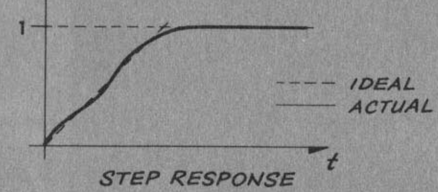
$$y = \frac{1}{T}\int_{t-T}^{t} x(t)\,dt = \frac{1 - e^{-Tp}}{Tp} \qquad (3\text{-}8)$$

where $T = \pi RC$. Figure (c) compares the actual response (to a step input) with the ideal.

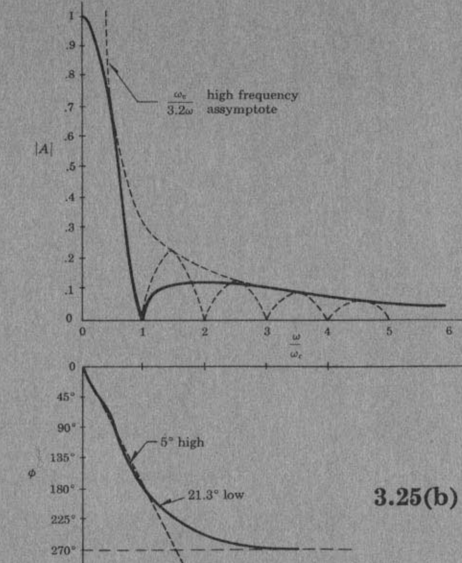
This filter is particularly useful for averaging functions of non-stationary random variables, since its transient-response time is minimum for a given averaging time.



3.25(a)



3.25(c)

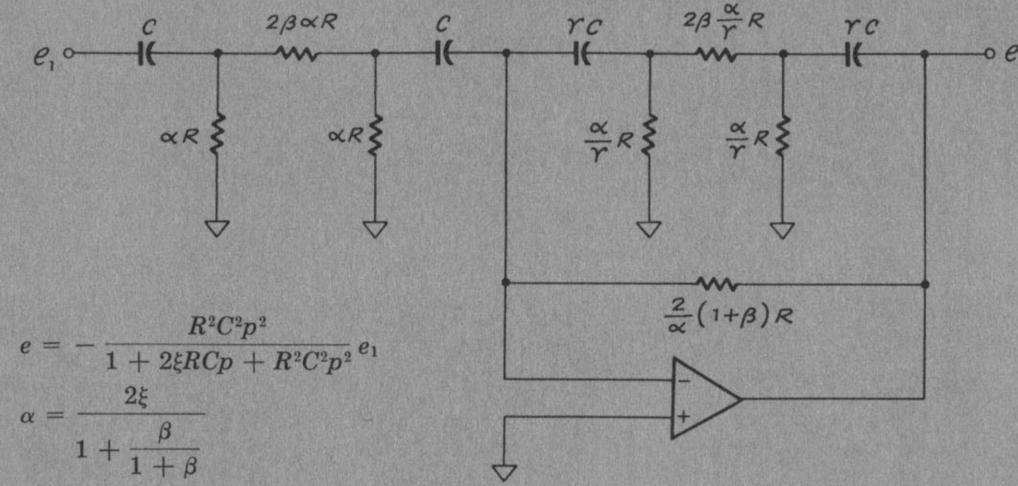


3.25(b)

III.25
II.1
to
II.4
III.22
III.23
III.24
III.26
III.27
III.28

III.26 HIGH-PASS FILTERS. By applying a conformal transformation to a low-pass filter characteristic it is possible to convert that characteristic to a high-pass, band-pass, or band-reject characteristic having exactly the same gain and phase as the low-pass at corresponding frequencies. (The chart of Section III.28 lists these transformations.)

The high-pass characteristic achieved by this conformal-transformation process can be very effective at removing all frequency components below the cut-off frequency, while passing without attenuation all frequencies above cut-off; however, if the signal has significant frequency content in the neighborhood of cut-off, the resulting transient response does not permit good reproduction of the original waveform, due to phase distortion. The filter producing the least phase shift at the cut-off frequency is the first-order lead circuit shown in II.16. Another effective high-pass circuit is shown in the figure. This can be used as the quadratic factor (second-order section) of high-order filters. Remember that the amplifier gain-bandwidth limitation ultimately limits the high-frequency pass band. Depending upon the amplifier used, small feedback capacitors may be necessary to achieve stability.



$$e = -\frac{R^2C^2p^2}{1 + 2\xi RCp + R^2C^2p^2}\,e_1$$

$$\alpha = \frac{2\xi}{1 + \frac{\beta}{1+\beta}}$$

$$\gamma = 1 - \frac{\frac{\beta}{1+\beta}}{\left(1 + \frac{\beta}{1+\beta}\right)^2}(2\xi)^2$$

β chosen to avoid capacitive load

make $\beta \ll 1$ to minimize noise gain

3.26

III.26
II.1
to
II.4
III.22
to
III.25
III.27
III.28

III.27 BAND-PASS AND BAND REJECT (NOTCH) FILTERS. A band-pass filter has two cut-off frequencies. Below the lower, ω_{CL}, and above the upper, ω_{CU}, signal components are highly attenuated. Between the two cut-off frequencies, the signal components are passed with nearly unity gain, and with phase shift varying significantly with frequency. To simplify calculation, it is convenient to define two terms:

$$\omega_0 = \sqrt{\omega_{CU}\omega_{CL}}\,, \text{ ``center frequency''} \qquad (3\text{-}9)$$

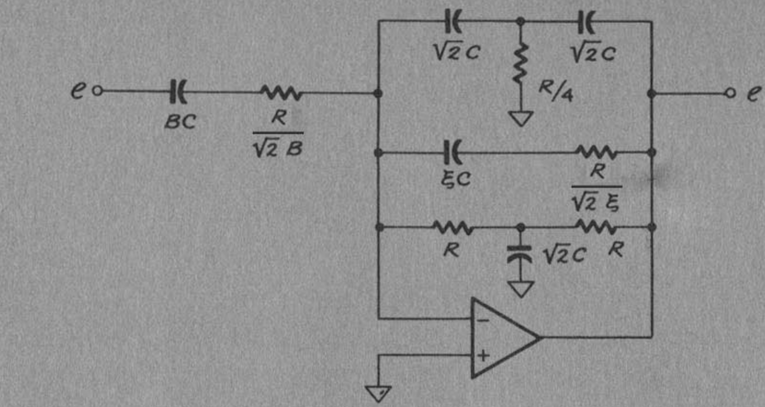
$$B = \frac{\omega_{CU} - \omega_{CL}}{2\omega_0}\,, \text{ ``normalized bandwidth''} \qquad (3\text{-}10)$$

We classify those band-pass filters having a $B \geq 1$ $(\omega_{CU}/\omega_{CL} > 6)$ as "wide-band" and those with $B < 1$ $(\omega_{CU}/\omega_{CL} < 6)$ as "narrow-band."

To get a wide-band-pass filter, we cascade a high-pass filter with cut-off at approximately ω_{CL} with a low-pass filter with cut-off at approximately ω_{CU}. The transformation listed in the Chart of III.28 yields a band-pass characteristic that is symmetrical about ω_0, when plotted on logarithmic coordinates. Circuit 3.23(a) is typical of the low-pass filters used, and circuit 3.26 is typical of the high-pass filters one would cascade with them, to achieve a wide-band-pass characteristic.

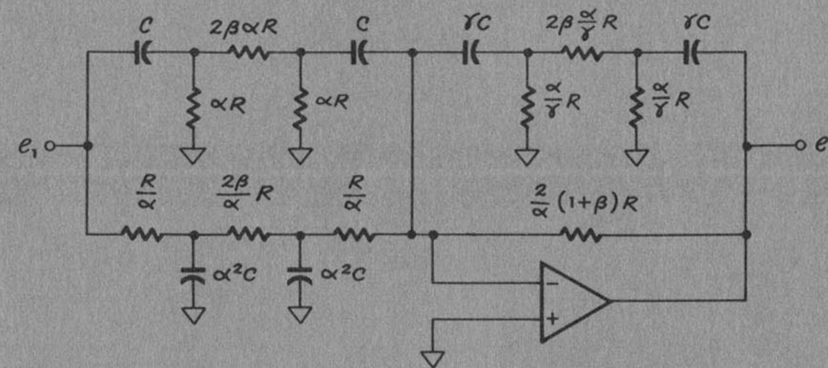
Narrow band-pass filters are used primarily for spectrum analysis. An active filter can be designed to respond to an extremely small band of frequencies, while rejecting all other components. When B, the normalized bandwidth, is very small, the damping ratio of each band-pass quadratic factor is also very small. This causes the filter to have a very slow transient response when a sudden change in spectral density of the input signal occurs. Circuit (a) is typical.

A band-reject filter, like the band-pass filter, has two cut-off frequencies. Between these two frequencies, ω_{CL} and ω_{CU}, signal components are heavily attenuated, whereas outside this band they are passed without significant attenuation. The same terms, ω_0 and B, are used in the design equations given in the Chart of III.28. Circuit (b) is typical; note the similarity to 3.26.



3.27(a)

$$e = \frac{2BRCp}{1 + 2\xi RCp + R^2C^2p^2}\, e_1$$



$$e = -\frac{1 + R^2C^2p^2}{1 + b_1RCp + b_2R^2C^2p^2}\, e_1$$

$$\gamma = b_2 - \frac{\dfrac{\beta}{1+\beta}}{\left(1 + \dfrac{\beta}{1+\beta}\right)^2} b_1^2 \qquad \mu = \frac{b_1}{1 + \dfrac{\beta}{1+\beta}}$$

β chosen to prevent capacitive load problems
(Make $\beta \ll 1$ to avoid noise gain problems)

3.27(b)

III.27
II.1
to
II.4
III.22
to
III.26
III.28

III.28
II.1 to II.4
II.15 to II.18
III.22 to III.27

III.28 FILTER DESIGN DATA. With Table (a), any of the first-order or second-order low-pass filter-circuits described and shown in III.23 may be used to predict the behavior of high-pass, band-pass, or band-reject filters of equivalent (conforming) design, as mentioned in III.26.

Table (b) summarizes the transfer functions of Butterworth and Paynter filters. For higher-order Butterworth factors, refer to III.24.

The following example illustrates the use of the chart:

Task: Design a band-pass filter, of the Butterworth Type, having a $\frac{1}{2}$ octave effective noise bandwidth centered at 1 radian/second (ω_0), to be used for spectral power density measurement. The filter is to have a very sharp cut-off, such that at $\omega = \sqrt{2}$ radians/second (the next highest center frequency in a comb filter), the gain is less than 0.1.

Procedure

Step 1. Determine B

Let $\omega_{CU} - \omega_{CL} = 3$ db bandwidth.

$$B \equiv \frac{\omega_{CU} - \omega_{CL}}{2\omega_0} \quad , \quad \omega \equiv \sqrt{\omega_{CU}\ \omega_{CL}} = 1 \text{ rad/sec}$$

Let noise bandwidth $b_N = \omega_{CU}' - \omega_{CL}'$

such that $\omega_0 = \sqrt{\omega_{CU}' - \omega_{CL}'} = 1$ rad/sec

For a half octave noise bandwidth: $\frac{\omega_{CU}'}{\omega_{CL}'} = \sqrt{2}$

Hence: $\frac{\omega_{CU}'}{\omega_0} = \frac{\omega_0}{\omega_{CL}'} = \sqrt[4]{2}$

Call: $K = \frac{2\pi b_N}{\omega_{CU} - \omega_{CL}} = \frac{\text{noise bandwidth}}{\text{3db bandwidth}}$

Values of K are presented in Table 3.

$$K = \frac{\omega_{CU}' - \omega_{CL}'}{\omega_{CU} - \omega_{CL}} = \frac{\frac{\omega_{CU}'}{\omega_0} - \frac{\omega_{CL}'}{\omega_0}}{2\frac{\omega_{CU} - \omega_{CL}}{2\omega_0}} = \frac{\sqrt[4]{2} - \frac{1}{\sqrt[4]{2}}}{2B}$$

Hence: $B = \frac{\sqrt[4]{2} - \frac{1}{\sqrt[4]{2}}}{2K} = \frac{.174}{K}$.

3.28 (a)

Step 2. Determine attenuation at $\omega = \sqrt{2}$ rad/sec using table (a)

Low Pass	Band Pass
$\frac{\omega_l}{\omega_c}$	$= \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\frac{1}{2B}$
	$= \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\frac{1}{2B} = \frac{\sqrt{2}}{4B}$

From table (b) the gain of the low-pass Butterworth filter is:

$$|A|^2 = \frac{1}{1 + \left(\frac{\omega_l}{\omega_c}\right)^{2n}} = \frac{1}{1 + \left(\frac{\sqrt{2}}{4B}\right)^{2n}}$$

Using the result of step 1 and values of K from the second L.E. filter article (Vol. 13, No. 3 & 4)

Order, $(2n)$	K	B	$\lvert A\rvert$
2	1.571	.111	.30
4	1.111	.157	.20
6	1.047	.166	.10 ← use

Step 3. Determine the transfer functions. Transforming the 3rd Order low-pass Butterworth Table (b) model using Table (a) gives:

$$e = \left[\frac{2B\frac{p}{\omega_0}}{1 + 2B\frac{p}{\omega_0} + \left(\frac{p}{\omega_0}\right)^2}\right]\left[\frac{2B\frac{p}{\alpha\omega_0}}{1 + 2\xi\frac{p}{\alpha\omega_0} + \left(\frac{p}{\alpha\omega_0}\right)^2}\right]\left[\frac{2B\frac{\alpha p}{\omega_0}}{1 + 2\xi\frac{\alpha P}{\omega_0} + \left(\frac{\alpha P}{\omega_0}\right)^2}\right]e_1$$

where: $\omega_0 = 1$ rad/sec
$B = .166$

$$\alpha = \frac{1 + \frac{B}{2}\sqrt{1 - \xi_L^2}}{1 - \frac{B}{2}\sqrt{1 - \xi_L^2}} = 1.155 \ (*)$$

$$\xi = \frac{2\xi_L B}{\alpha + \frac{1}{\alpha}} = .0822 \quad \text{(See Table a)}$$

$\xi_L = .5$ (See Table b)

(*) This approximation of the α definition equation, Table (a), can be used when $B < \frac{1}{2}$.

3.28 (b)

The circuit of Figure 3.27(a) can be used for each of the quadratic factors. Arbitrarily we can select the largest capacitor, $\sqrt{2}C$, to be a convenient value, 1 μF.

Table a. TRANSFORMATIONS OF LOW PASS FILTERS

LOW PASS	HIGH PASS	BAND PASS	BAND REJECT
Mapping Relation			
$\frac{p_l}{\omega_c} =$	$\frac{\omega_0}{p}$	$\left(\frac{p}{\omega_0} + \frac{\omega_0}{p}\right)\frac{1}{2B}$	$\frac{2B}{\left(\frac{p}{\omega_0} + \frac{\omega_0}{p}\right)}$
Corresponding Frequencies			
$\frac{\omega_l}{\omega_c} =$	$-\frac{\omega_0}{\omega}$	$\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\frac{1}{2B}$	$\frac{-2B}{\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$
Transfer Function First Order Model			
$\frac{1}{1 + \frac{p_l}{\omega_L}} \rightarrow$	$\frac{\frac{p}{\omega_0'}}{1 + \frac{p}{\omega_0'}}$	$\frac{2B_1\frac{p}{\omega_0}}{1 + 2B_1\frac{p}{\omega_0} + \left(\frac{p}{\omega_0}\right)^2}$	$\frac{1 + \left(\frac{p}{\omega_0}\right)^2}{1 + 2B_1\frac{p}{\omega_0} + \left(\frac{p}{\omega_0}\right)^2}$
Second Order Model			
$\frac{1}{1 + 2\xi_L\frac{p_l}{\omega_L} + \left(\frac{p_l}{\omega_L}\right)^2} \rightarrow$	$\frac{\left(\frac{p}{\omega_0'}\right)^2}{1 + 2\xi_L\frac{p}{\omega_0'} + \left(\frac{p}{\omega_0'}\right)^2}$	$\left[\frac{2B_1\left(\frac{p}{\alpha\omega_0}\right)}{1 + 2\xi\left(\frac{p}{\alpha\omega_0}\right) + \left(\frac{p}{\alpha\omega_0}\right)^2}\right]\left[\frac{2B_1\left(\frac{\alpha p}{\omega_0}\right)}{1 + 2\xi\left(\frac{\alpha p}{\omega_0}\right) + \left(\frac{\alpha p}{\omega_0}\right)^2}\right]$	$\left[\frac{1 + \left(\frac{p}{\omega_0}\right)^2}{1 + 2\xi\left(\frac{p}{\alpha\omega_0}\right) + \left(\frac{p}{\alpha\omega_0}\right)^2}\right]\left[\frac{1 + \left(\frac{p}{\omega_0}\right)^2}{1 + 2\xi\left(\frac{\alpha p}{\omega_0}\right) + \left(\frac{\alpha p}{\omega_0}\right)^2}\right]$
Definitions			
ω_c cut off frequency	$\omega_0' = \frac{\omega_c}{\omega_L}\omega_0$	$B_1 = B\frac{\omega_L}{\omega_c}$	$B_1 = B\frac{\omega_c}{\omega_L}$
ω_L natural frequency of 2nd order factor		$\xi = \frac{2\xi_L B_1}{\alpha + \frac{1}{\alpha}}$	
ξ_L damping ratio; p_l low pass Heaviside operator		$\alpha = \sqrt{\frac{B_1^2 + 1 + \sqrt{B_1^4 + 2(1 - 2\xi_L^2)B_1^2 + 1}}{2}} + \sqrt{\frac{B_1^2 - 1 + \sqrt{B_1^4 + 2(1 - 2\xi_L^2)B_1^2 + 1}}{2}}$	
ω_l low pass radian frequency		$B = \frac{\omega_{CU} - \omega_{CL}}{2\omega_0}$, $= \sqrt{\omega_{CU}\omega_{CL}}$	

Table b. LOW PASS TRANSFER FUNCTIONS

ORDER	BUTTERWORTH	PAYNTER
1.	$\frac{1}{\left(1 + \frac{p}{\omega_c}\right)}$	
2.	$\frac{1}{\left(1 + \sqrt{2}\frac{p}{\omega_c} + \left(\frac{p}{\omega_c}\right)^2\right)}$	$\frac{1}{\left(1 + 3\frac{p}{\omega_c} + 4\left(\frac{p}{\omega_c}\right)^2\right)}$
3.	$\frac{1}{\left(1 + \frac{p}{\omega_c}\right)\left(1 + \frac{p}{\omega_c} + \left(\frac{p}{\omega_c}\right)^2\right)}$	$\frac{1}{\left(1 + 2\frac{p}{\omega_c}\right)\left(1 + 1.2\left(\frac{p}{\omega_c}\right) + 1.6\left(\frac{p}{\omega_c}\right)^2\right)}$
	$\omega_c = 3$ db frequency; $\lvert A\rvert = .7$ when $\omega = \omega_c$; $\lvert A\rvert^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}$	$\phi_{lag} \approx \pi\frac{\omega}{\omega_c}$

3.28(c)