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Dynamic Angle Estimation with Inertial MEMS

Analog Devices

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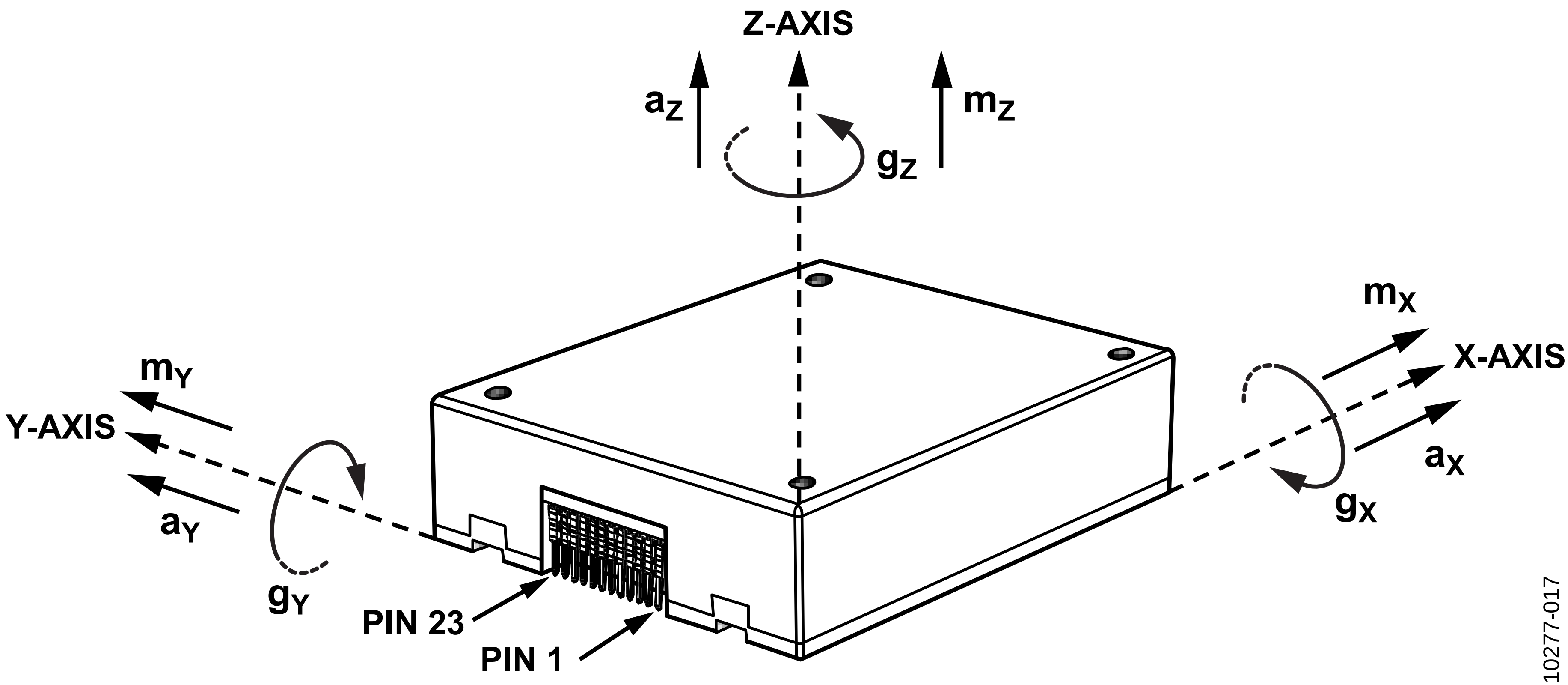
Agenda

- Sensor to angle basics
- Accelerometer basics
- Accelerometer behaviors
- Gyroscope basics
- Gyroscope behaviors
- Key factors for successful integration

Sensor to Angle Basics: 2+ axes

Gyro/Accel: pitch & roll, yaw = gyro only



Accelerometer Basics

Accelerometer Basics

Ideal Relationship

$$y(t) = K \cdot a(t)$$

Sensor output (LSB)

Ideal Sensitivity (LSB per g)

Acceleration (g)

Simplified Errors

$$y(t) = K \cdot (1 + \varepsilon_S) \cdot a(t) + \varepsilon_B$$

Motion-Dependent Errors

Motion-Independent Errors

“Motion” Dependent vs Independent

$$y(t) = K \cdot (1 + \varepsilon_S) \cdot a(t) + \varepsilon_B$$

ACCELEROMETERS	Each axis				
Dynamic Range		±5			g
Initial Sensitivity	See Table 16 for data format	0.2475	0.25	0.2525	mg/LSB
Repeatability[1]	$-40°C \le T_A \le +70°C$			1	%
Sensitivity Temperature Coefficient	$-40°C \le T_A \le +70°C$		±40		ppm/°C
Misalignment	Axis to axis		±0.2		Degrees
	Axis to frame (package)		±0.5		Degrees
Nonlinearity	Best fit straight line		±0.2		% of FS
Bias Repeatability[1, 2]	$-40°C \le T_A \le +70°C$, 1 σ		±8		mg
In-Run Bias Stability	1 σ, SMPL_PRD = 0x0001		0.075		mg
Velocity Random Walk	1 σ, SMPL_PRD = 0x0001		0.073		m/sec/√hr
Bias Temperature Coefficient	$-40°C \le T_A \le +85°C$		±0.04		mg/°C
Bias Supply Sensitivity	+3.15 V ≤ VDD ≤ +3.45 V		1.5		mg/V
Output Noise	No filtering		2.25		mg rms
Noise Density	No filtering		0.105		mg/√Hz rms
−3 dB Bandwidth			330		Hz
Sensor Resonant Frequency			5.5		kHz

“Motion” definition implies representation of non-zero orientation

Accelerometers
Motion Independent Behaviors

Bias Repeatability

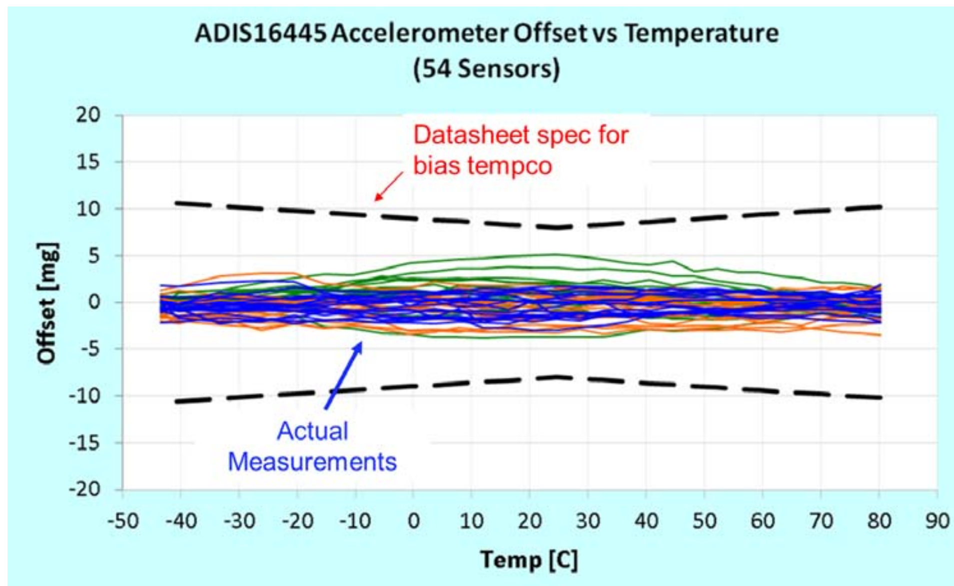
- Specification approach varies quite a lot, across the industry
- Long-term predictions would seem to be valuable
- Changes in mechanical stress & aging profile can impact this
  - How well is the MEMS structure protected from the changing forces associated with humidity, temperature cycling, etc.
- Has most impact when the incline angle is 0 degrees
- For example, if the bias repeatability is equal to 8mg, the resulting angle error is 0.46 degrees:

$$\theta_x = a\sin\left({0.008g}/{1g} \right) = 0.46^\circ$$

- Mitigation (sensor-only) is through a four-point tumble test

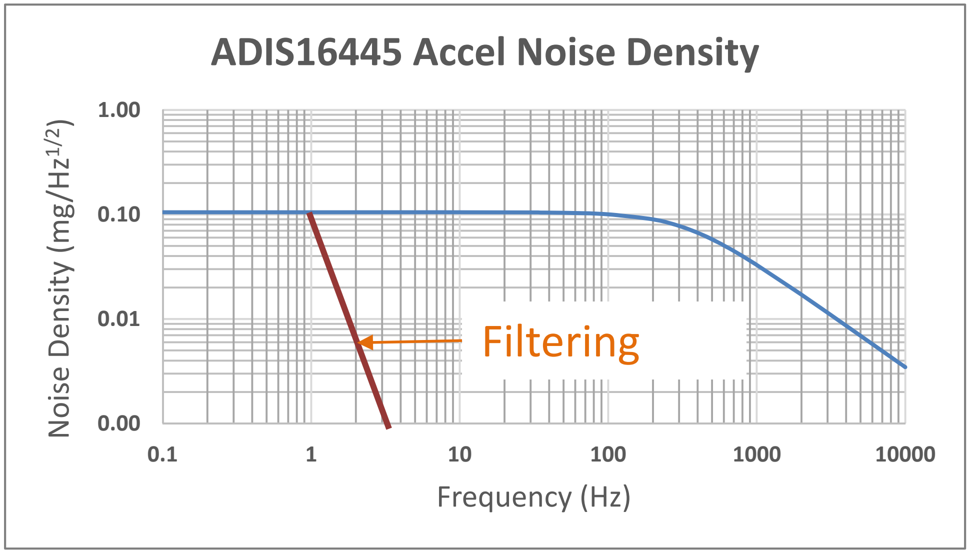
Bias vs. Temperature

- Bias dependence on temperature is often captured as “Bias Temperature Coefficient” in the specification table.
- Residual errors typically are not linear when the behaviors are calibrated
- KEY: do not apply “linear” assumptions to this behavior over entire temperature range
- Example:
  - Bias tempco = 0.04mg/°C
  - Temperature change = 20°C
  - Bias change = 0.8mg
  - Angle error = 0.05 deg



Noise

- Total noise = Noise Density x Noise Bandwidth$^{1/2}$
- For example:
  - Noise density = 0.105 mg/Hz$^{1/2}$
  - Assume a single-pole, low-pass filter, where 2Hz is the cut-off frequency
  - Total noise = 0.000105 x (1.57 x 2)$^{1/2}$ = 0.186ug
  - Angle_noise = asin(0.000186) = ~0.011 deg
  - KEY = sensor noise is not the dominant error source for a 0.5° goal



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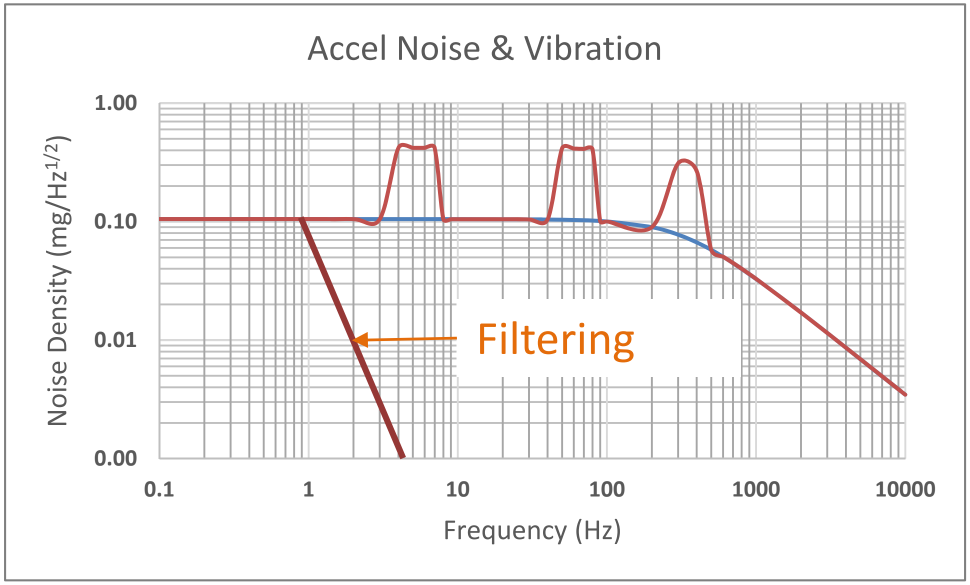
Vibration Response

- Vibration can be the primary driver for filtering
- Proper sample rates enable digital filtering help address these artifacts.
- Post filtering, residual error is related to linearity
  - If this was ~200µg/g^2:
  - At 4gms: Residual bias = 4^2 x 0.0002 = 0.0032g rms -> 0.18 deg



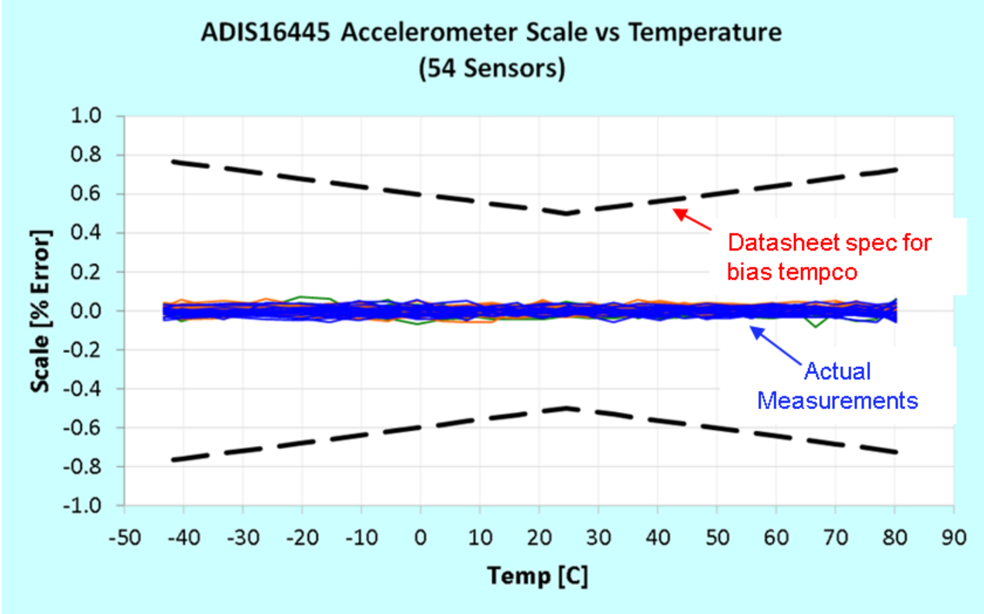
Accelerometers
Motion Dependent Behaviors

Sensitivity repeatability

- Specification approach varies quite a lot, across the industry
- Long-term predictions would seem to be valuable
- Package stress & aging profile can impact this
  - How well is the MEMS structure protected from the changing forces associated with humidity, temperature cycling, etc.
- Example impact:
  - Tilt = 30 degrees
  - Ideal accelerometer output = 1g x sin(30) = 0.5g
  - Add 1% of sensitivity error: 1.01 x 0.5 = 0.505
  - Calculate the angle: asin(0.505) = 30.3
  - <u>Error = 0.3 degrees</u>

Sensitivity (Scale) vs. Temperature

- Bias dependence on temperature is often captured as "Sensitivity Temperature Coefficient" in the specification table.
- Residual errors typically are not linear when the behaviors are calibrated
- KEY: do not apply "linear" assumptions to this behavior over entire temperature range
- Example:
  - Sensitivity tempco = 40ppm/℃
  - Temperature change = 20℃
  - Bias change = 800ppm = 0.08%
  - <u>Much lower than repeatability</u>



Gyroscope Basics

Gyroscope Basics

Ideal Relationship

$$y(t) = K \cdot \omega(t)$$

Sensor output (LSB)

Ideal Sensitivity (LSB per °/sec)

Rate of Rotation (°/sec)

Simplified Errors

$$y(t) = K \cdot (1 + \varepsilon_S) \cdot \omega(t) + \varepsilon_B$$

Motion-Dependent Errors

Motion-Independent Errors

"Motion" Dependent vs Independent

$$y(t) = K \cdot (1 + \varepsilon_S) \cdot \omega(t) + \varepsilon_B$$

Parameter	Test Conditions/Comments	Min	Typ	Max	Unit
GYROSCOPES					
Dynamic Range		±250			°/sec
Initial Sensitivity	±250°/sec, see Table 12		0.01		°/sec/LSB
	±125°/sec		0.005		°/sec/LSB
	±62°/sec		0.0025		°/sec/LSB
Repeatability	$-40°C \le T_A \le +70°C$			1	%
Sensitivity Temperature Coefficient	$-40°C \le T_A \le +70°C$		±40		ppm/°C
Misalignment	Axis to axis		±0.05		Degrees
	Axis to frame (package)		±0.5		Degrees
Nonlinearity	Best fit straight line		±0.1		% of FS
Bias Repeatability[1,]	$-40°C \le T_A \le +70°C$, 1 σ		0.5		°/sec
In-Run Bias Stability	1 σ, SMPL_PRD = 0x0001		12		°/hr
Angular Random Walk	1 σ, SMPL_PRD = 0x0001		0.56		°/√hr
Bias Temperature Coefficient	$-40°C \le T_A \le +85°C$		±0.005		°/sec/°C
Linear Acceleration Effect on Bias	Any axis, 1 σ		±0.015		°/sec/g
Bias Supply Sensitivity	+3.15 V ≤ VDD ≤ +3.45 V		±0.2		°/sec/V
Output Noise	±250°/sec range, no filtering		0.22		°/sec rms
Rate Noise Density	f = 25 Hz, ±250°/sec range, no filtering		0.011		°/sec/√Hz rms
−3 dB Bandwidth			330		Hz
Sensor Resonant Frequency			17.5		kHz

"Motion" definition implies representation of non-zero orientation

Gyroscopes
Motion Independent Behaviors

Bias Repeatability

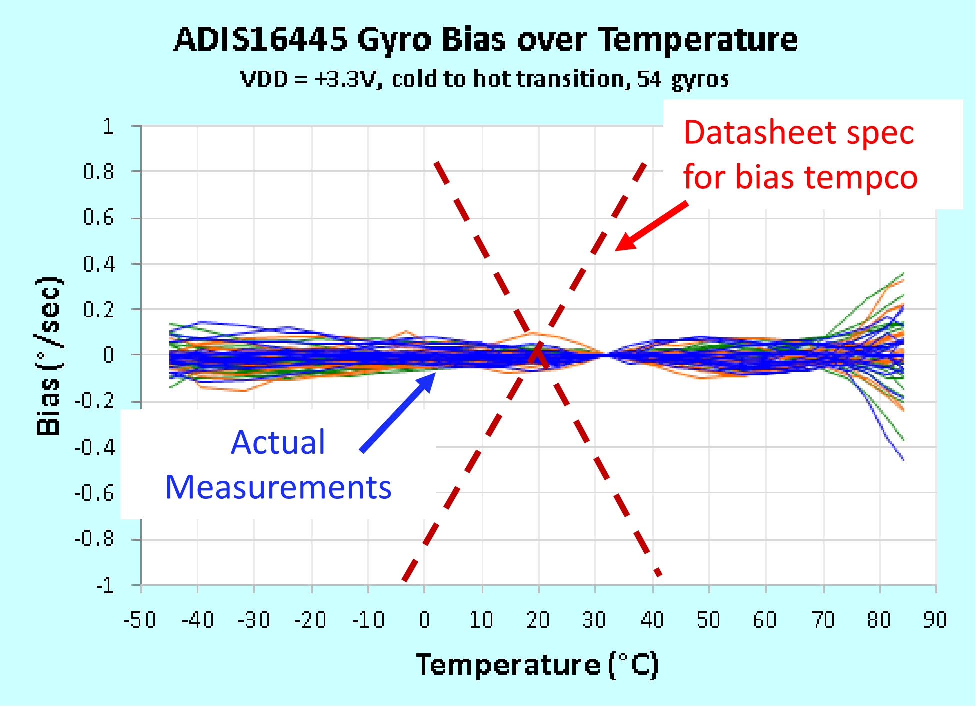
- Specification approach varies quite a lot across the industry
- Long-term predictions would seem to be valuable
- Changes in mechanical stress & aging profile can impact this
  - How well is the MEMS structure protected from the changing forces associated with humidity, temperature cycling, etc.
- The bias level will translate into angle drift over time

$$\theta_x = 0.2 \,{}^{\circ}\!/_{\sec}$$

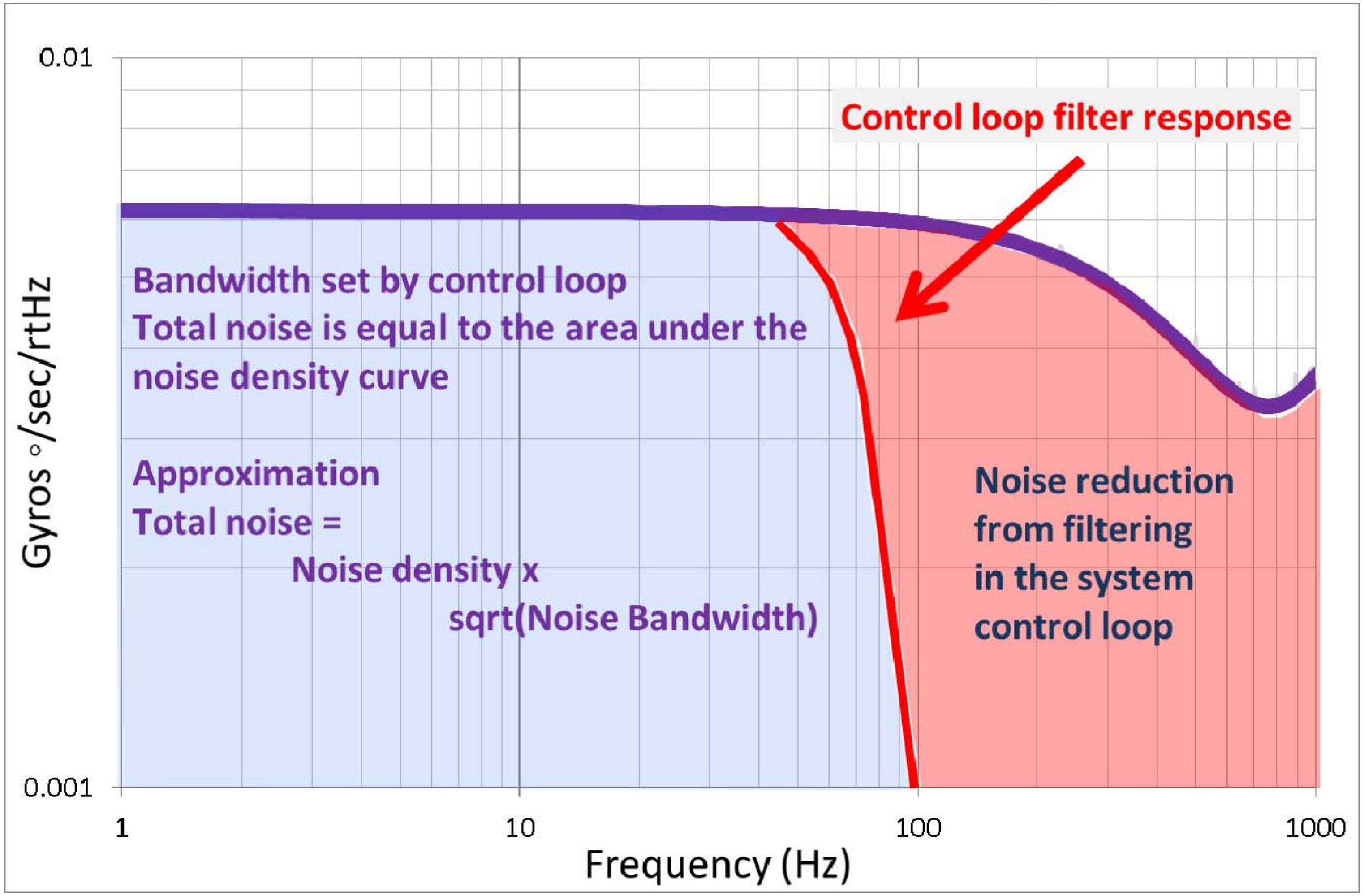
- This can be managed through periodic observation

Bias Temperature Coefficient

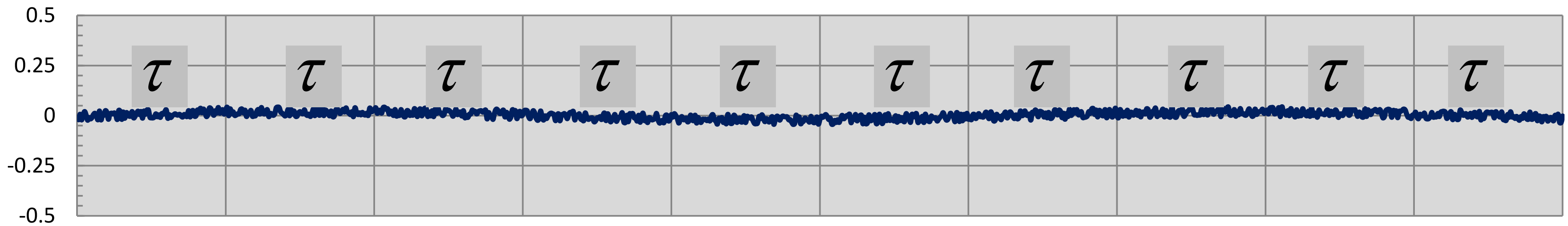
- Bias dependence on temperature is often captured as "Bias Temperature Coefficient" in specification table.
- Residual errors typically are not linear when the behaviors are calibrated
- KEY: do not apply "linear" assumptions to this behavior over entire temperature range
- Example:
  - Bias tempco = 0.04 °/sec/°C
  - Temperature change = 2°C
  - Bias change = 0.08 °/sec

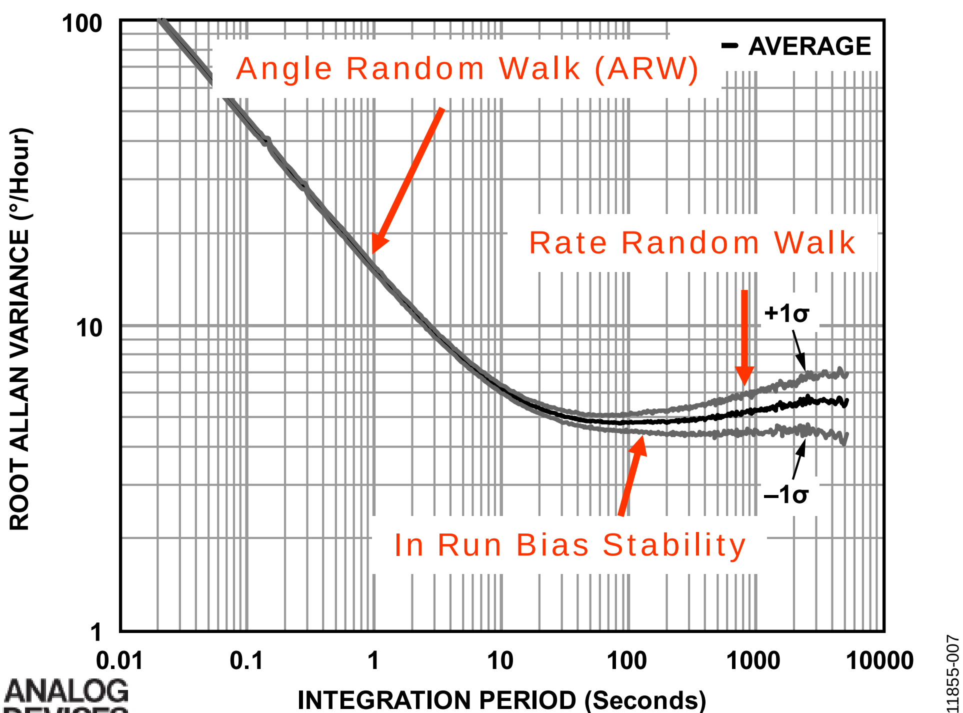


Noise vs Frequency



Allan Variance - stability vs time





- Take at least 30, sequential time records, using sufficient sample rate, and manage temp, supply and vibration influences.
- The total time of each record represents the integration time on the AVAR plot (τ)
- Take the average of each time record.
- Use the following variance equation:

$$AVAR = \sqrt{\frac{1}{2\times\left(N_R - 1\right)} \times \sum_{n=1}^{N_R} \left(X_{n+1} - X_n\right)^2}$$

n = record number

X_n = average of record "n"

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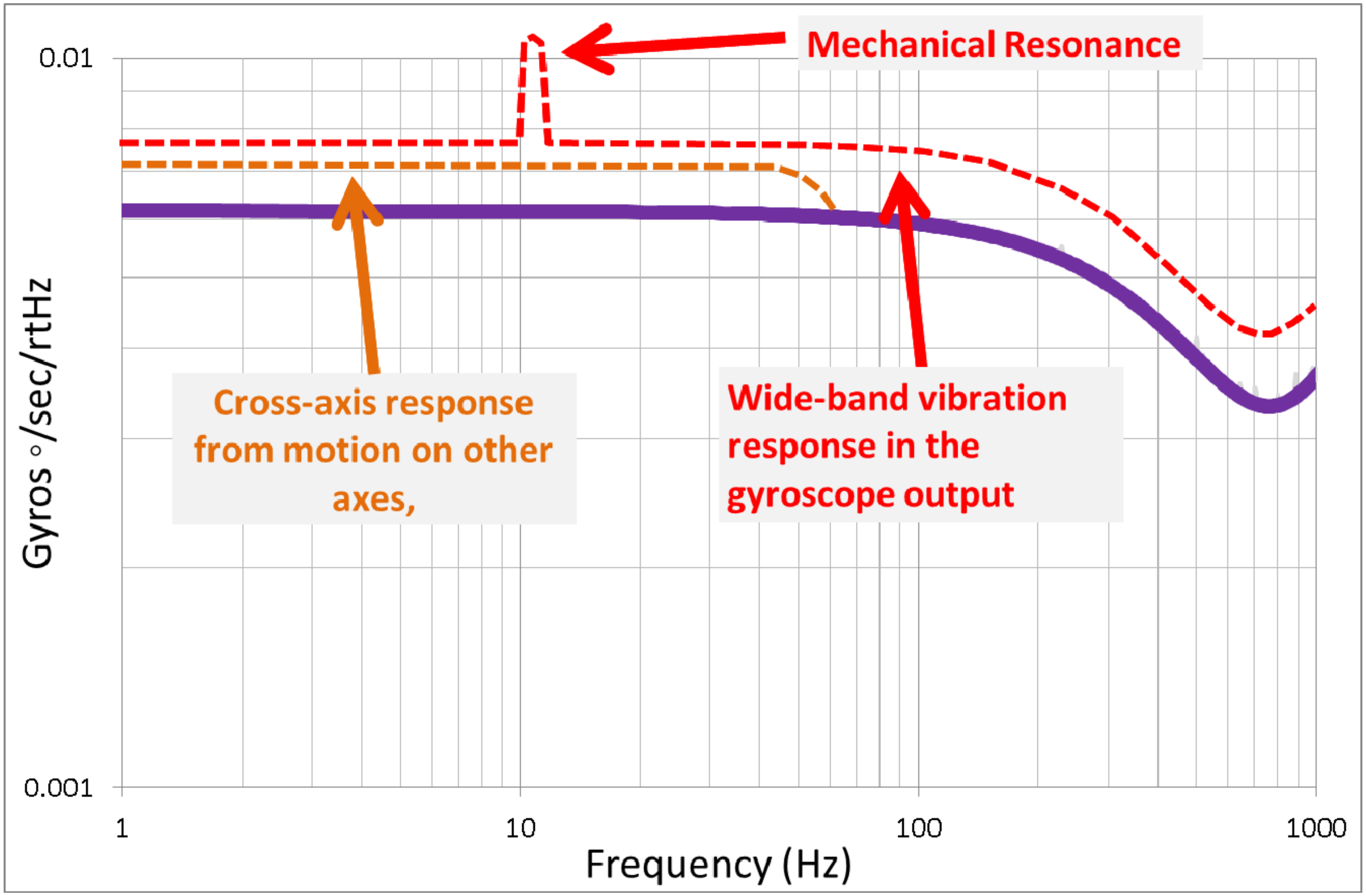
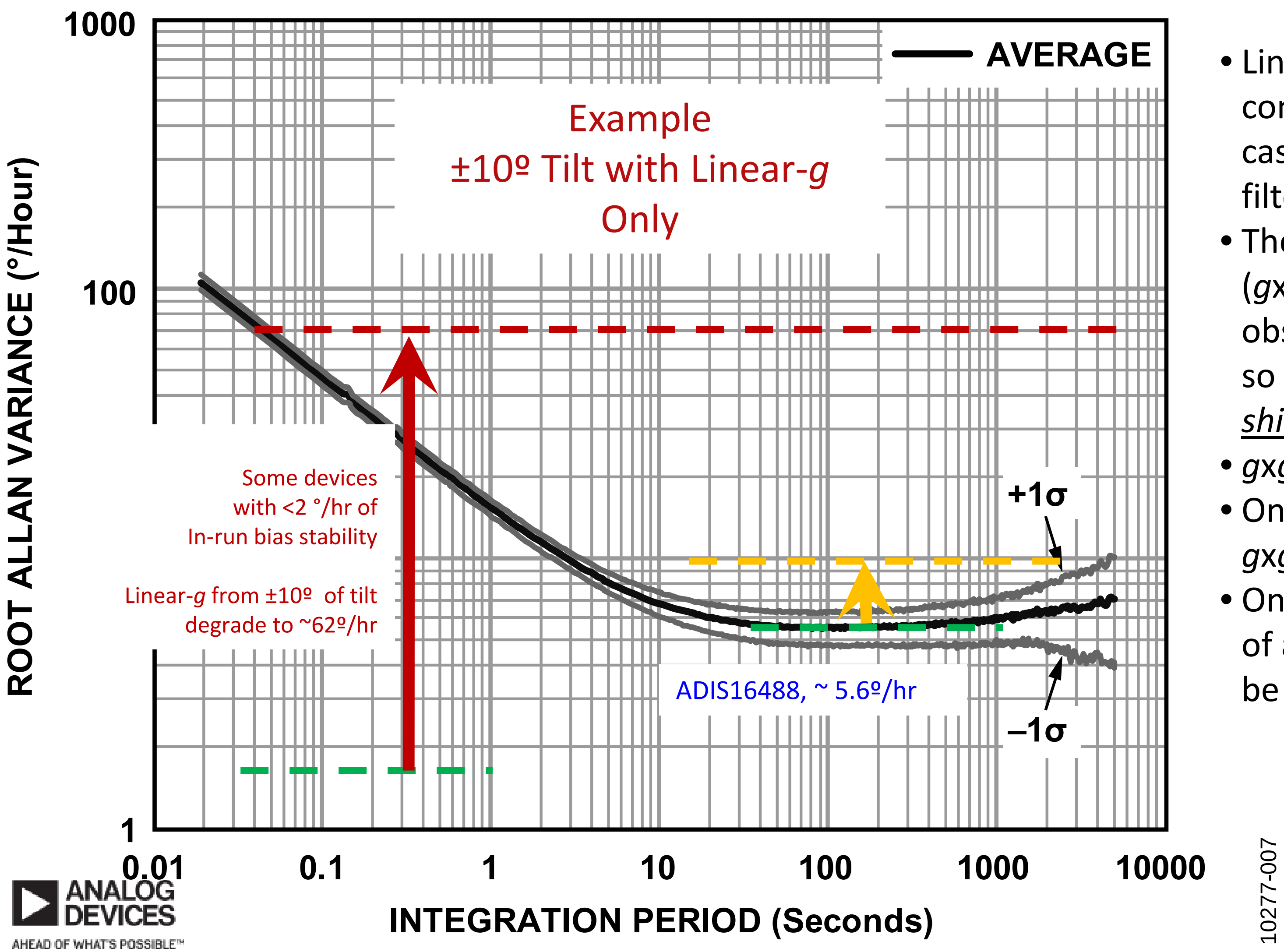
Gyroscope
Motion Independent Behaviors
Performance Spoilers

Vibration/Cross-axis Spectral View



Allan Variance w/ vibration/cross-axis



- Linear-*g* can be observed, compensated and in some cases, removed with filtering.
- The issue with rectified (*gxg*) is that it is difficult to observe and is one-sided, so filtering results in a *bias shift*.
- *gxg* is rarely specified
- One gyro's performance for *gxg* is 0.005°/sec/g^2
- On this product, the impact of a 2*g* rms vibration would be 72°/hour!

Gyroscopes
Motion Dependent Behaviors

Sensitivity repeatability

- Specification approach varies quite a lot in the industry
- Long-term predictions would seem to be valuable
- Package stress & aging profile can impact this
  - How well is the MEMS structure protected from the changing forces associated with humidity, temperature cycling, etc.
- Example impact:
  - Error in angle ~ error in sensitivity
  - 10 degree change in angle
  - Error = 1% of 10 degrees = 0.1 degrees

Sensitivity Temperature Coefficient

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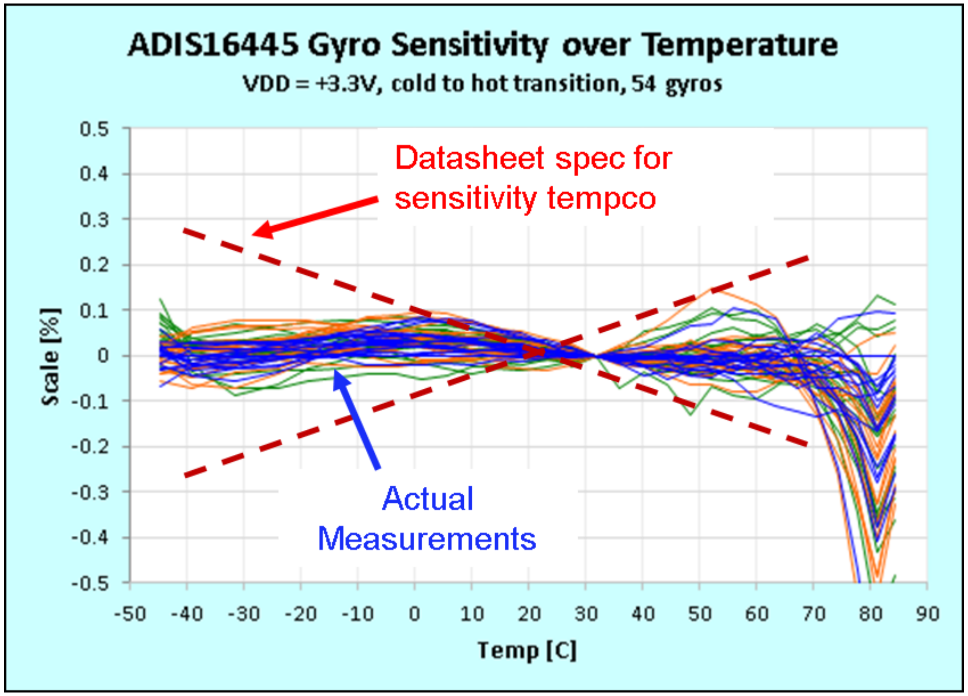

Thank you for listening…

Questions?

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