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Low-Pass to Band-Pass Filter Transformation

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IN THIS MINI TUTORIAL

A transformation algorithm is available for converting low-pass poles into equivalent band-pass poles. This is one in a series of mini tutorials describing discrete circuits for op amps.

INTRODUCTION

Band-pass filters can be classified as either wide band or narrow band, depending on the separation of the poles. If the corner frequencies of the band-pass are widely separated (by more than 2 octaves), the filter is wide band and is made up of separate low-pass and high-pass sections, which will be cascaded. This mini tutorial focuses on narrow band filters.

The assumption made is that with the widely separated poles, interaction between them is minimal. This condition does not hold in the case of a narrow-band band-pass filter, where the separation is less than 2 octaves.

Typically, filters are described using the low-pass prototype because the low-pass configuration is the standard. To transform the filter into a band-pass, start with the complex pole pairs of the low-pass prototype, α and β. The pole pairs are known to be complex conjugates. This implies symmetry around dc (0 Hz.). The process of transformation to the band-pass case is one of mirroring the response around dc of the low-pass prototype to the same response around the new center frequency, F_0.

This clearly implies that the number of poles and zeros is doubled when the band-pass transformation is done. As in the low-pass case, the poles and zeros below the real axis are ignored. Thus, an nth order low-pass prototype transforms into an nth order band-pass, even though the filter order will be 2n. An nth order band-pass filter consists of n sections vs. n/2 sections for the low-pass prototype. It may be convenient to think of the response as n poles up and n poles down.

The value of Q_{BP} is determined by

$$Q_{BP} = \frac{F_0}{BW} \tag{1}$$

where *BW* is the bandwidth at some level, typically −3 dB.

A TRANSFORMATION ALGORITHM

A transformation algorithm was defined by Geffe (see the References section) for converting low-pass poles into equivalent band-pass poles.

Given the pole locations of the low-pass prototype

$$-\alpha \pm j\beta \tag{2}$$

and the values of F_0 and Q_{BP}, the following calculations result in two sets of values for Q and frequencies, F_H and F_L, which define a pair of band-pass filter sections.

$$C = \alpha^2 + \beta^2 \tag{3}$$

$$D = \frac{2\alpha}{Q^{BP}} \tag{4}$$

$$E = \frac{C}{{Q_{BP}}^2} + 4 \tag{5}$$

$$G = \sqrt{E^2 - 4D^2} \tag{6}$$

$$Q = \sqrt{\frac{E+G}{2D^2}} \tag{7}$$

Observe that the Q of each section will be the same.

The pole frequencies are determined by

$$M = \frac{\alpha Q}{Q_{BP}} \tag{8}$$

$$W = M + \sqrt{M^2 - 1} \tag{9}$$

$$F_{BP1} = \frac{F_0}{W} \tag{10}$$

$$F_{BP2} = W\ F_0 \tag{11}$$

Each pole pair transformation will also result in 2 zeros that will be located at the origin.

A normalized low-pass real pole with a magnitude of α_0 is transformed into a band-pass section where

$$Q = \frac{Q_{BP}}{\alpha_0} \tag{12}$$

and the frequency is F_0.

Each single pole transformation also results in a zero at the origin.

Elliptical function low-pass prototypes contain zeros as well as poles. In transforming the filter, the zeros must be transformed as well. Given the low-pass zeros at $\pm j\omega_Z$, the band-pass zeros are obtained as follows:

$$M = \frac{\alpha Q}{Q_{BP}} \quad (13)$$

$$W = M + \sqrt{M^2 - 1} \quad (14)$$

$$F_{BP1} = \frac{F_0}{W} \quad (15)$$

$$F_{BP2} = W\, F_0 \quad (16)$$

Since the gain of a band-pass filter peaks at F_{BP} instead of F_0, an adjustment in the amplitude function is required to normalize the response of the aggregate filter. The gain of the individual filter section is given by:

$$A_R = A_0 \sqrt{1 + Q^2 \left(\frac{F_0}{F_{BP}} - \frac{F_{BP}}{F_0} \right)^2} \quad (17)$$

where:
A_0 = gain a filter center frequency
A_R = filter section gain at resonance
F_0 = filter center frequency
F_{BP} = filter section resonant frequency

The low-pass prototype is now converted to a band-pass filter. The equation string outlined above is used for the transformation. Each pole of the prototype filter transforms into a pole pair. Therefore, the 3-pole prototype, when transformed, will have 6 poles (3 pole pairs). In addition, there will be 6 zeros at the origin.

POLE LOCATIONS

The pole locations for the LP prototype were taken from the design table (see MT-206). They are outlined in Table 1.

Table 1.

Stage	α	β	F_0	α
1	0.2683	0.8753	1.0688	0.5861
2	0.5366		0.6265	

The first stage is the pole pair and the second stage is the single pole. Note the unfortunate convention of using α for two entirely separate parameters. The α and β on the left are the pole locations in the s-plane. These are the values used in the transformation algorithms. The α on the right is 1/Q, which is what the design equations for the physical filters want to see.

Part of the transformation process is to specify the 3 dB bandwidth of the resultant filter. In this case, this bandwidth will be set to 500 Hz. The results of the transformation yield results as shown in

Table 2.

Stage	F_0	Q	A_0
1	804.5	7.63	3.49
2	1243	7.63	3.49
3	1000	3.73	1

The reason for the gain requirement for the first two stages is that their center frequencies will be attenuated relative to the center frequency of the total filter. Since the resultant Qs are moderate (less than 20) the multiple feedback topology will be chosen (see MT-218).

Figure 1 is the schematic of the filter and Figure 2 shows the frequency response.

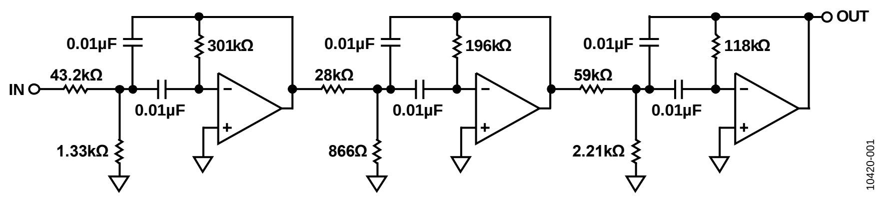

Figure 1. Band-Pass Transformation

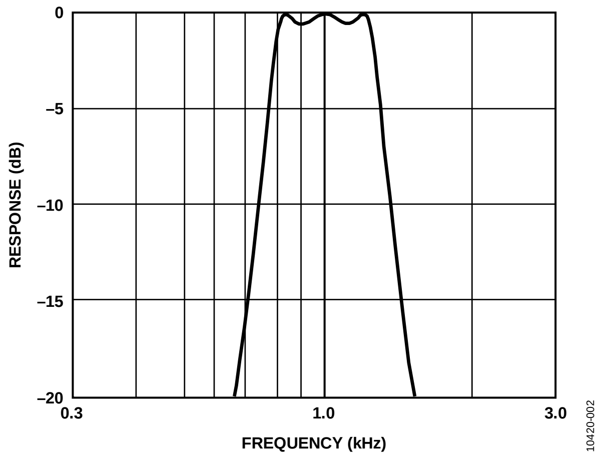

Figure 2. Band-Pass Filter Response

Note that again there is symmetry around the center frequency. In addition, the 500 Hz bandwidth is not 250 Hz either side of the center frequency (arithmetic symmetry). Instead, the symmetry is geometric, which means that any two frequencies (F_1 and F_2) of equal amplitude are related by

$$F_0 = \sqrt{F_1 \times F2} \quad (18)$$

REFERENCES

Geffe, P. R. "Designer's Guide to Active Band-Pass Filters," EDN, Apr. 5 1974, pp. 46-52.

Zumbahlen, Hank. *Linear Circuit Design Handbook*. Elsevier. 2008. ISBN: 978-7506-8703-4.

REVISION HISTORY

3/12—Revision 0: Initial Version