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Mini Tutorial
MT-204

# The Bessel Response

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## IN THIS MINI TUTORIAL

*The Bessel filter, the filter of choice for op amps if phase response is of concern, is one of multiple discrete circuits described in a series of mini tutorials.*

## INTRODUCTION TO THE BESSEL RESPONSE

Butterworth filters have fairly good amplitude and transient behavior. The Chebyshev filters improve on the amplitude response at the expense of transient behavior. The Bessel filter is optimized to obtain better transient response due to a linear phase (that is, constant delay) in the pass band. This means that there will be relatively poorer frequency response (less amplitude discrimination).

The Bessel filter is designed for linear phase. Another way to say this is that there is constant delay in the pass band. The step response shows no overshoot and there is no ringing. The impulse response shows no ringing as well. Thus, the Bessel filter is the filter of choice if phase response is the primary concern.

The poles of the Bessel filter can be determined by locating all of the poles on a circle and separating their imaginary parts by

$$\frac{2}{n}$$

where $n$ is the number of poles.

Note that the top and bottom poles are distanced by where the circle crosses the jω axis by

$$\frac{1}{n}$$

or half the distance between the other poles.

Figure 1 shows the pole locations for a 5-pole Bessel filter.

## BESSEL POLE LOCATIONS

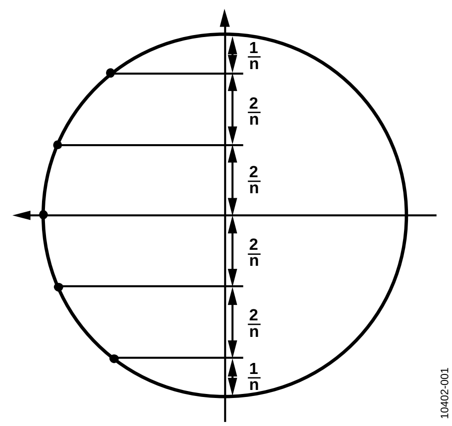


*Figure 1. Bessel Pole Locations*

The frequency response, group delay, impulse response and step response for the Bessel filters are cataloged in Figure 2 through Figure 6.

The pole locations and corresponding $\omega_o$ and $\alpha$ terms for the Bessel filter are tabulated in Figure 7.

## BESSEL RESPONSE

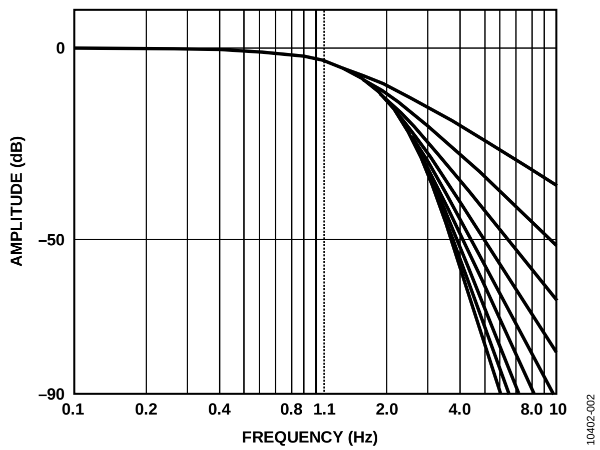


*Figure 2. Bessel Response, Amplitude*

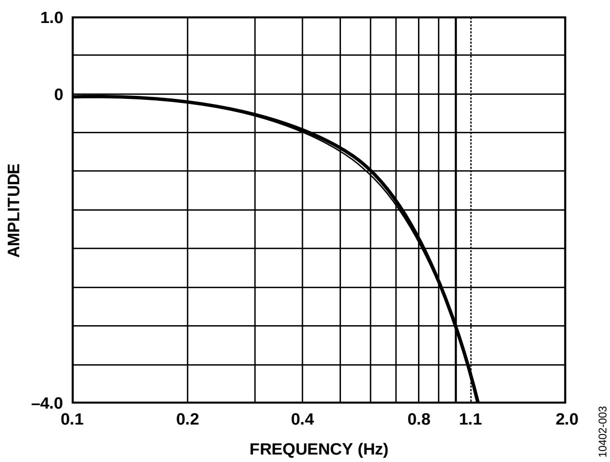


*Figure 3. Bessel Response, Amplitude (Detail)*

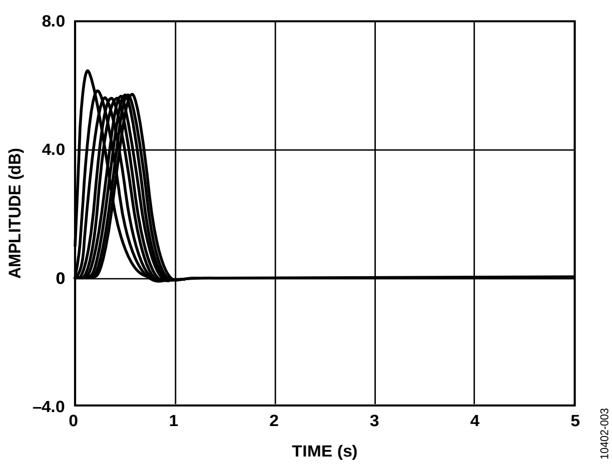


*Figure 4. Bessel Response, Impulse Response*

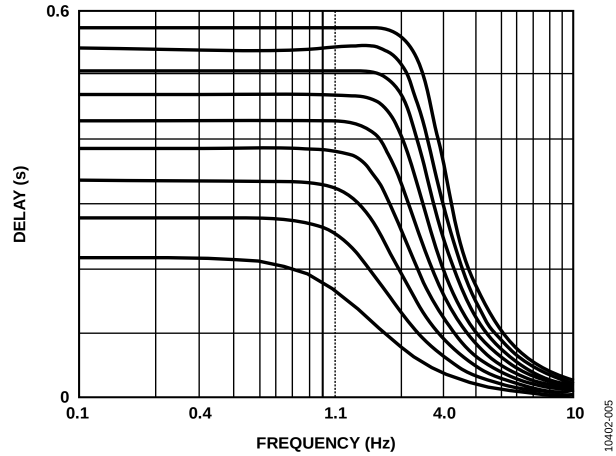


*Figure 5. Bessel Response, Group Delay*

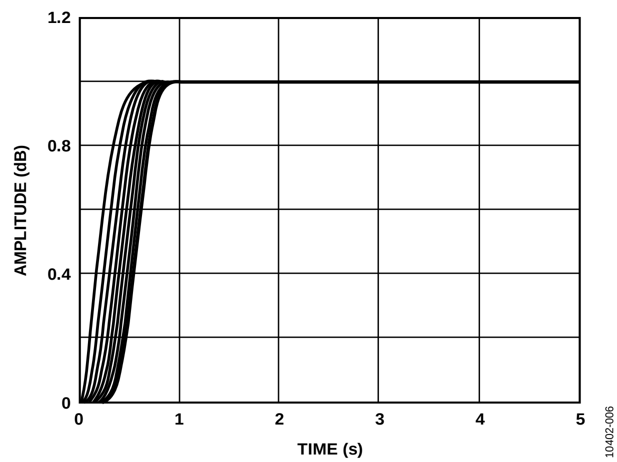


*Figure 6. Bessel Response, Step Response*

## BESSEL TABLE DESIGN

**Figure 7. Bessel Design Table**

| Order | Section | Real Part | Imaginary Part | $F_0$ | α | Q | −3 dB Frequency | Peaking Frequency | Peaking Level |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 1 | 1.1050 | 0.6368 | 1.2754 | 1.7328 | 0.5771 | 1.0020 | | |
| 3 | 1 | 1.0509 | 1.0025 | 1.4524 | 1.4471 | 0.6910 | 1.4185 | | |
| | 2 | 1.3270 | | 1.3270 | | | 1.3270 | | |
| 4 | 1 | 1.3596 | 0.4071 | 1.4192 | 1.9160 | 0.5219 | 0.9705 | | |
| | 2 | 0.9877 | 1.2476 | 1.5912 | 1.2414 | 0.8055 | | 0.7622 | 0.2349 |
| 5 | 1 | 1.3851 | 0.7201 | 1.5611 | 1.7745 | 0.5635 | 1.1876 | | |
| | 2 | 0.9606 | 1.4756 | 1.7607 | 1.0911 | 0.9165 | | 1.1201 | 0.7768 |
| | 3 | 1.5069 | | 1.5069 | | | 1.5069 | | |
| 6 | 1 | 1.5735 | 0.3213 | 1.6060 | 1.9596 | 0.5103 | 1.0638 | | |
| | 2 | 1.3836 | 0.9727 | 1.6913 | 1.6361 | 0.6112 | 1.4323 | | |
| | 3 | 0.9318 | 1.6640 | 1.9071 | 0.9772 | 1.0234 | | 1.3786 | 1.3851 |
| 7 | 1 | 1.6130 | 0.5896 | 1.7174 | 1.8784 | 0.5324 | 1.2074 | | |
| | 2 | 1.3797 | 1.1923 | 1.8235 | 1.5132 | 0.6608 | 1.6964 | | |
| | 3 | 0.9104 | 1.8375 | 2.0507 | 0.8879 | 1.1262 | | 1.5961 | 1.9860 |
| | 4 | 1.6853 | | 1.6853 | | | 1.6853 | | |
| 8 | 1 | 1.7627 | 0.2737 | 1.7838 | 1.9763 | 0.5060 | 1.1675 | | |
| | 2 | 0.8955 | 20044 | 2.1953 | 0.8158 | 1.2258 | | 1.7932 | 2.5585 |
| | 3 | 1.3780 | 1.3926 | 1.9591 | 1.4067 | 0.7109 | | 0.2011 | 0.0005 |
| | 4 | 1.6419 | 0.8256 | 1.8378 | 1.7868 | 0.5597 | 1.3849 | | |
| 9 | 1 | 1.8081 | 0.5126 | 1.8794 | 1.9242 | 0.5197 | 1.2774 | | |
| | 2 | 1.6532 | 1.0319 | 1.9488 | 1.6966 | 0.5894 | 1.5747 | | |
| | 3 | 1.3683 | 1.5685 | 2.0815 | 1.3148 | 0.7606 | | 0.7668 | 0.0807 |
| | 4 | 0.8788 | 2.1509 | 2.3235 | 0.7564 | 1.3220 | | 1.9632 | 30949 |
| | 5 | 1.8575 | | 1.8575 | | | 1.8575 | | |
| 10 | 1 | 1.9335 | 0.2451 | 1.9490 | 1.9841 | 0.5040 | 1.2685 | | |
| | 2 | 1.8467 | 0.7335 | 1.9870 | 1.8587 | 0.5380 | 1.4177 | | |
| | 3 | 1.6661 | 1.2246 | 2.0678 | 1.6115 | 0.6205 | 1.7848 | | |
| | 4 | 1.3648 | 1.7395 | 2.2110 | 1.2346 | 0.8100 | | 1.0785 | 0.2531 |
| | 5 | 0.8686 | 2.2994 | 2.4580 | 0.7067 | 1.4150 | | 2.1291 | 3.5944 |

## REFERENCES

Zumbahlen, Hank. *Linear Circuit Design Handbook*. Elsevier. 2008. ISBN: 978-7506-8703-4.

## REVISION HISTORY

**2/12—Revision 0: Initial Version**