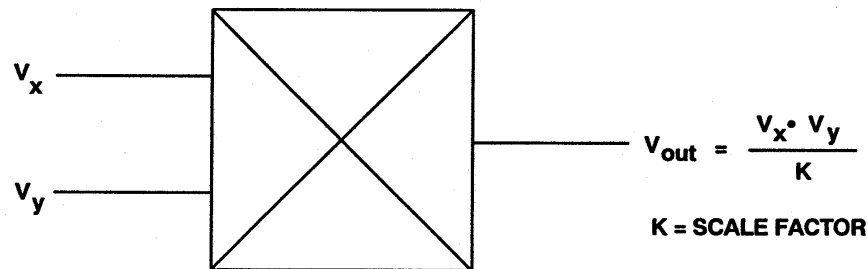


Analog Multipliers

ANALOG MULTIPLIER BASICS

An analog multiplier is a device having two input ports and an output port. The signal at the output is the product of the two input signals. If both input and output signals are voltages, the transfer characteristic is the product of the two voltages divided by a scaling factor, K , which has the dimension of voltage as shown in Figure 1.



Type	V_x	V_y	V_{out}
Single Quadrant	Unipolar	Unipolar	Unipolar
Two Quadrant	Bipolar	Unipolar	Bipolar
Four Quadrant	Bipolar	Bipolar	Bipolar

Figure 1: Basic Analog Multiplier and Definition of Multiplier Quadrants

From a mathematical point of view, multiplication is a "four quadrant" operation—that is to say that both inputs may be either positive or negative, as may be the output. Some of the circuits used to produce electronic multipliers, however, are limited to signals of one polarity. If both signals must be unipolar, we have a "single quadrant" multiplier, and the output will also be unipolar. If one of the signals is unipolar, but the other may have either polarity, the multiplier is a "two quadrant" multiplier, and the output may have either polarity (and is "bipolar"). The circuitry used to produce one- and two-quadrant multipliers may be simpler than that required for four quadrant multipliers, and since there are many applications where full four quadrant multiplication is not required, it is common to find accurate devices which work only in one or two quadrants. An example is the [AD539](#), a wideband dual two-quadrant multiplier which has a single unipolar V_y input with a relatively limited bandwidth of 5 MHz, and two bipolar V_x inputs, one per multiplier, with bandwidths of 60 MHz. A block diagram of the AD539 is shown in Figure 2.

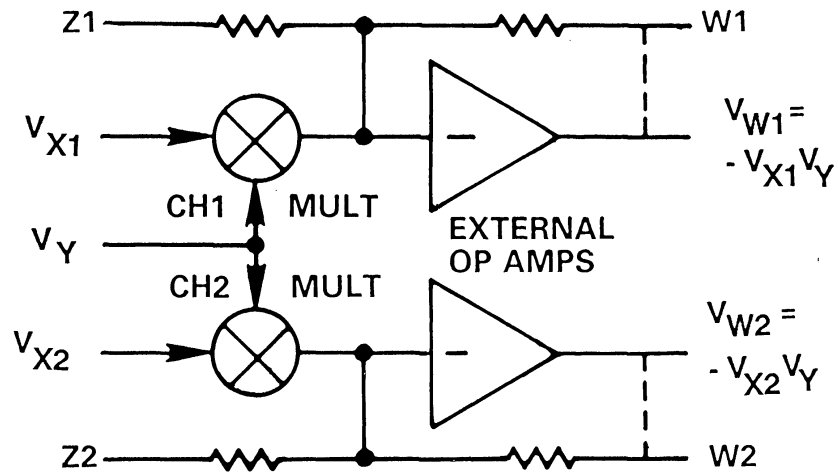


Figure 2: AD539 Analog Multiplier Block Diagram

The simplest electronic multipliers use logarithmic amplifiers. The computation relies on the fact that the antilog of the sum of the logs of two numbers is the product of those numbers as shown in Figure 3.

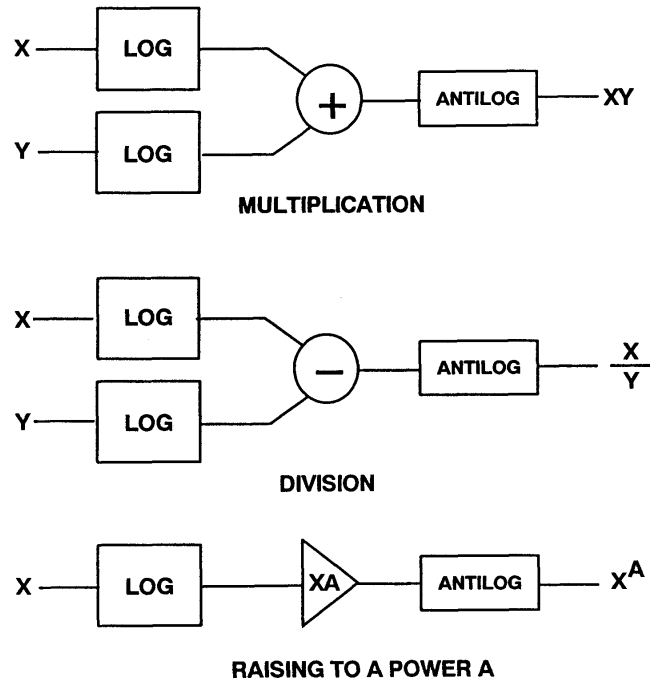


Figure 3: Multiplication Using Log Amps

The disadvantages of this type of multiplication are the very limited bandwidth and single quadrant operation. A far better type of multiplier uses the "Gilbert Cell". This structure was invented by Barrie Gilbert in the late 1960s. (See References 1 and 2).

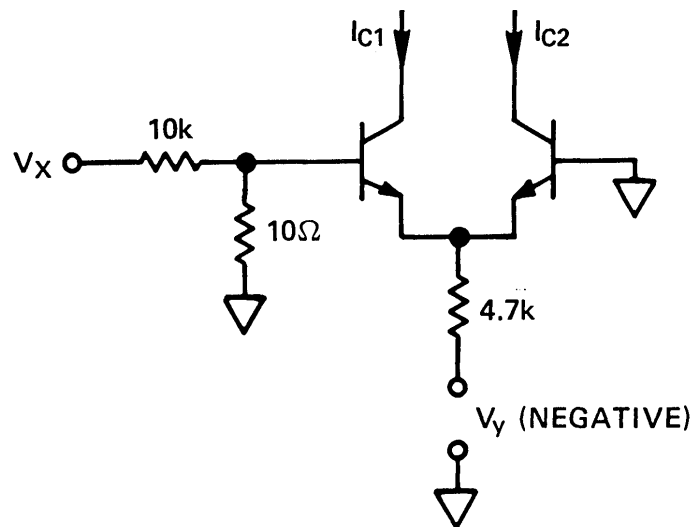
GILBERT CELL MULTIPLIER

There is a linear relationship between the collector current of a silicon junction transistor and its transconductance (gain) which is given by

$$dI_C / dV_{BE} = qI_C / kT, \text{ where} \tag{Eq. 1}$$

I_C = the collector current, V_{BE} = the base-emitter voltage, q = the electron charge (1.60219×10^{-19}), k = Boltzmann's constant (1.38062×10^{-23}), T = the absolute temperature.

This relationship may be exploited to construct a multiplier with a long-tailed pair of silicon transistors, as shown in Figure 4.



$$I_{c1} - I_{c2} = \Delta I_C = \frac{q}{kT} \left(\frac{V_y + V_{be}}{4.7 \times 10^3} \right) \left(\frac{10}{10,010} \right) V_x$$

$$= 8.3 \times 10^{-6} (V_y + 0.6) V_x \text{ @ } 25^\circ\text{C}$$

Figure 4: Basic Transconductance Multiplier

This is a rather poor multiplier because (1) the Y input is offset by the V_{BE} which changes non-linearly with V_Y ; (2) the X input is non-linear as a result of the exponential relationship between I_C and V_{BE} ; and (3) the scale factor varies with temperature.

Gilbert realized that this circuit could be linearized and made temperature stable by working with currents, rather than voltages, and by exploiting the logarithmic I_C/V_{BE} properties of transistors as shown in Figure 5. The X input to the Gilbert Cell takes the form of a differential current, and the Y input is a unipolar current. The differential X currents flow in two diode-connected transistors, and the logarithmic voltages compensate for the exponential V_{BE}/I_C relationship. Furthermore, the q/kT scale factors cancel. This gives the Gilbert Cell the linear transfer function

$$\Delta I_C = \frac{\Delta I_X I_Y}{I_X} \tag{Eq. 2}$$

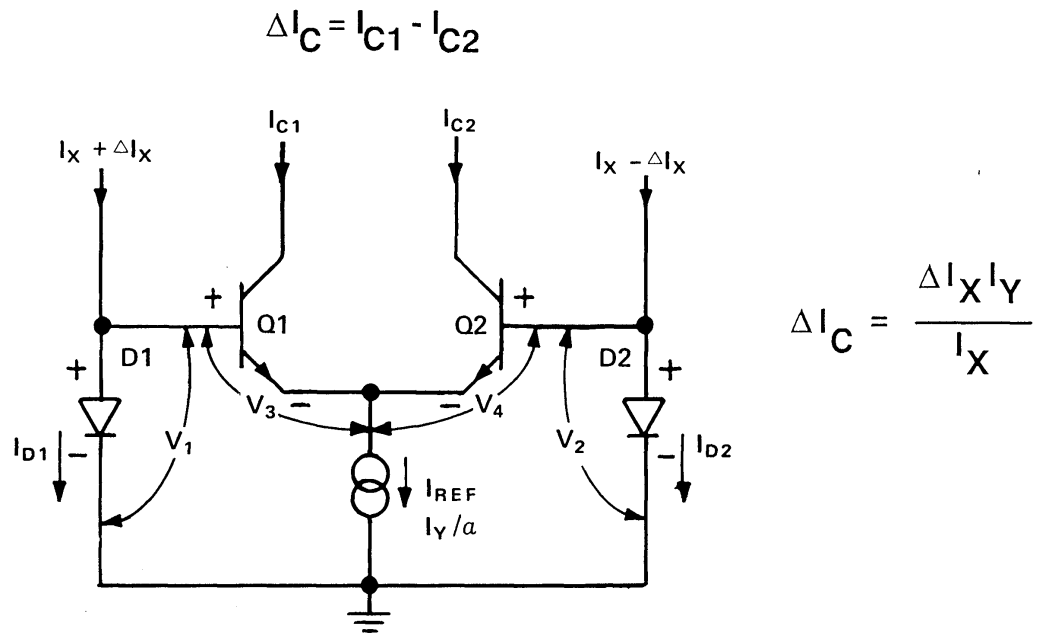


Figure 5: Basic Transconductance Multiplier

As it stands, the Gilbert Cell has three inconvenient features: (1) its X input is a differential current; (2) its output is a differential current; and (3) its Y input is a unipolar current—so the cell is only a two quadrant multiplier.

By cross-coupling two such cells and using two voltage-to-current converters as shown in Figure 6, we can convert the basic architecture to a four quadrant device with voltage inputs, such as the [AD534](#). At low and medium frequencies, a subtractor amplifier may be used to convert the differential current at the output to a voltage. Because of its voltage output architecture, the bandwidth of the AD534 is only about 1 MHz, although the [AD734](#), a later version, has a bandwidth of 10 MHz.

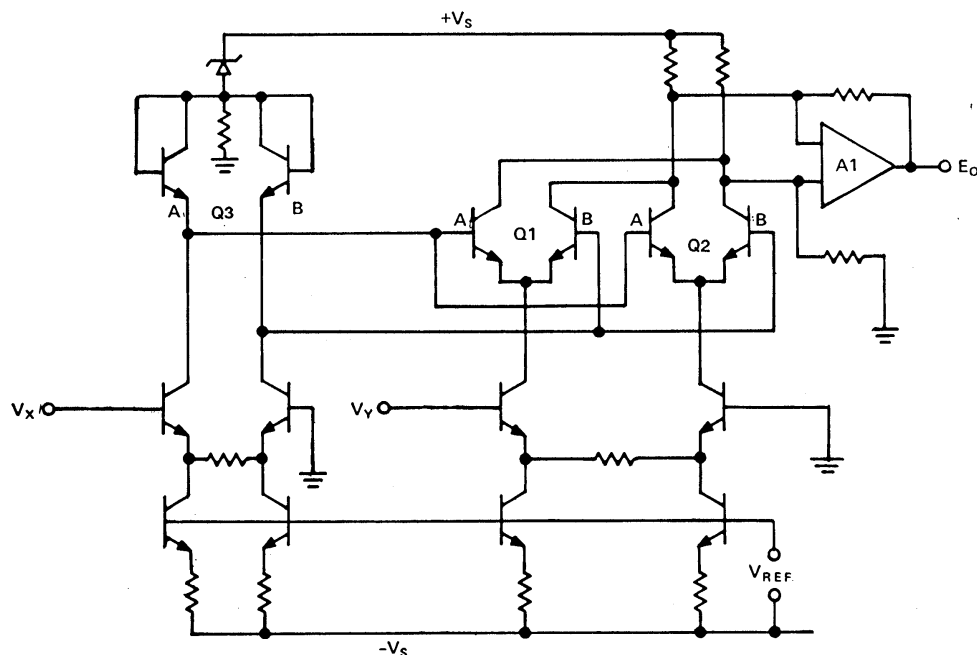


Figure 6: [AD534](#): A Four-Quadrant Translinear Multiplier

In Figure 6, Q1A & Q1B, and Q2A & Q2B form the two core long-tailed pairs of the two Gilbert cells, while Q3A and Q3B are the linearizing transistors for both cells. There is also an operational amplifier acting as a differential current to single-ended voltage converter, but for higher speed applications, the cross-coupled collectors of Q1 and Q2 form a differential open collector current output (as in the [AD834](#) 500 MHz multiplier).

The translinear multiplier relies on the matching of a number of transistors and currents. This is easily accomplished on a monolithic chip. Even the best IC processes have some residual errors, however, and these show up as four dc error terms in such multipliers (see Offset voltage on the X input shows up as feedthrough from the Y input. Offset voltage on the Y input shows up as feedthrough from the X input. Offset voltage on the Z input causes offset in the output signal, and resistor mismatch causes gain error.

In early Gilbert Cell multipliers, these errors had to be trimmed by means of resistors and potentiometers external to the chip, which was somewhat inconvenient. With modern analog processes, which permit the laser trimming of SiCr thin film resistors on the chip itself, it is possible to trim these errors during manufacture so that the final device has very high accuracy. Internal trimming has the additional advantage that it does not reduce the high frequency performance, as may be the case with external trim pots.

Because the internal structure of the translinear multiplier is necessarily differential, the inputs are usually differential as well (after all, if a single-ended input is required it is not hard to ground one of the inputs). This is not only convenient in allowing common-mode signals to be rejected, it also permits more complex computations to be performed. The [AD534](#) (shown

previously in Figure 6) is the classic example of a four-quadrant multiplier based on the Gilbert Cell. It has an accuracy of 0.1% in the multiplier mode, fully differential inputs, and a voltage output. However, as a result of its voltage output architecture, its bandwidth is only about 1 MHz.

For wideband applications, the basic multiplier with open collector current outputs is used. The [AD834](#) is an 8-pin device with differential X inputs, differential Y inputs, differential open collector current outputs, and a bandwidth of over 500 MHz. A block diagram is shown in Figure 7.

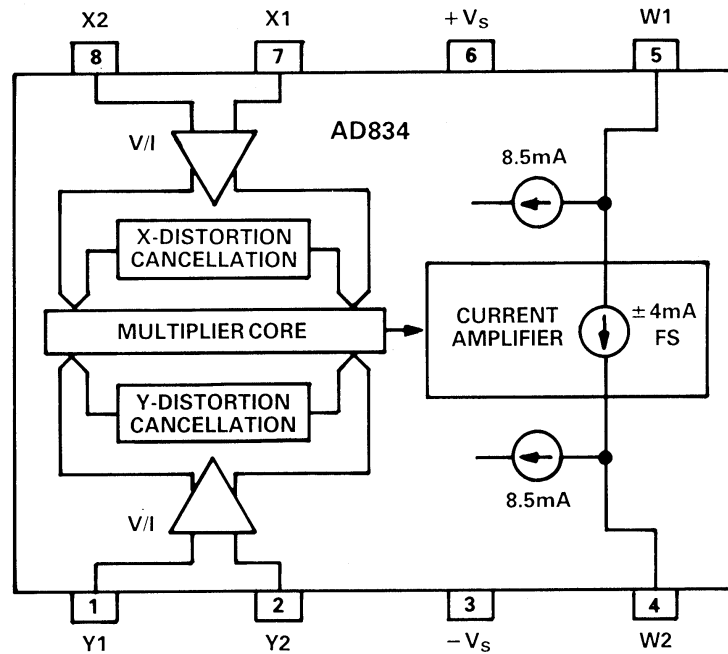


Figure 7: [AD834](#) 500MHz Four-Quadrant Multiplier

The AD834 is a true linear multiplier with a transfer function of

$$I_{OUT} = \frac{V_x \cdot V_y}{1V \cdot 250\Omega} \tag{Eq. 3}$$

Its X and Y offsets are trimmed to 500 μV (3 mV max), and it may be used in a wide variety of applications including multipliers (broadband and narrowband), squarers, frequency doublers, and high frequency power measurement circuits. A consideration when using the AD834 is that, because of its very wide bandwidth, its input bias currents, approximately 50 μA per input, must be considered in the design of input circuitry lest, flowing in source resistances, they give rise to unplanned offset voltages.

A basic wideband multiplier using the AD834 is shown in Figure 8. The differential output current flows in equal load resistors, R1 and R2, to give a differential voltage output. This is the simplest application circuit for the device. Where only the high frequency outputs are required, transformer coupling may be used, with either simple transformers or baluns.

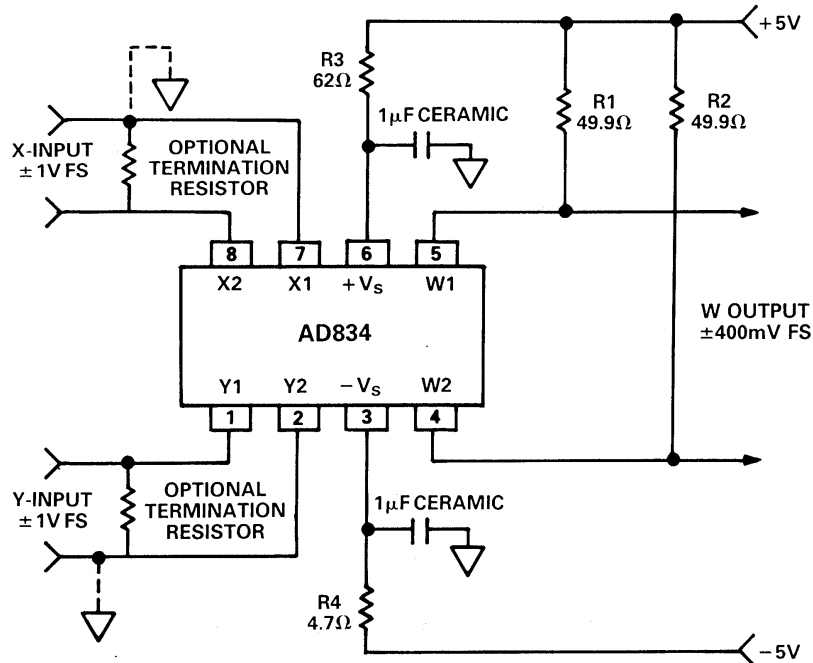
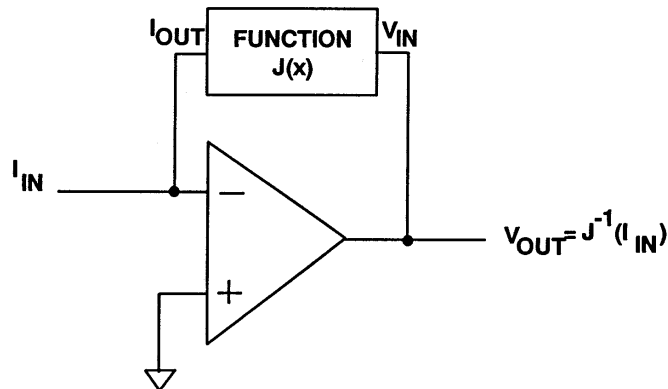


Figure 8: Basic Connections for the AD834

USING MULTIPLIERS WITH OP AMPS TO PERFORM ARITHMETIC FUNCTIONS

Multipliers can be placed in the feedback loop of op amps to form several useful functions. Figure 9 illustrates the basic principle of analog computation that a function generator in a negative feedback loop computes the inverse function (provided, of course, that the function is monotonic over the range of operations). Figure 10 shows a multiplier and an op amp configured as a divider in both inverting and non-inverting mode.



NOTE: FUNCTION MUST BE MONOTONIC OVER THE RELEVANT RANGE

Figure 9: A Function Generator in a Negative Feedback Loop Generates the Inverse Function

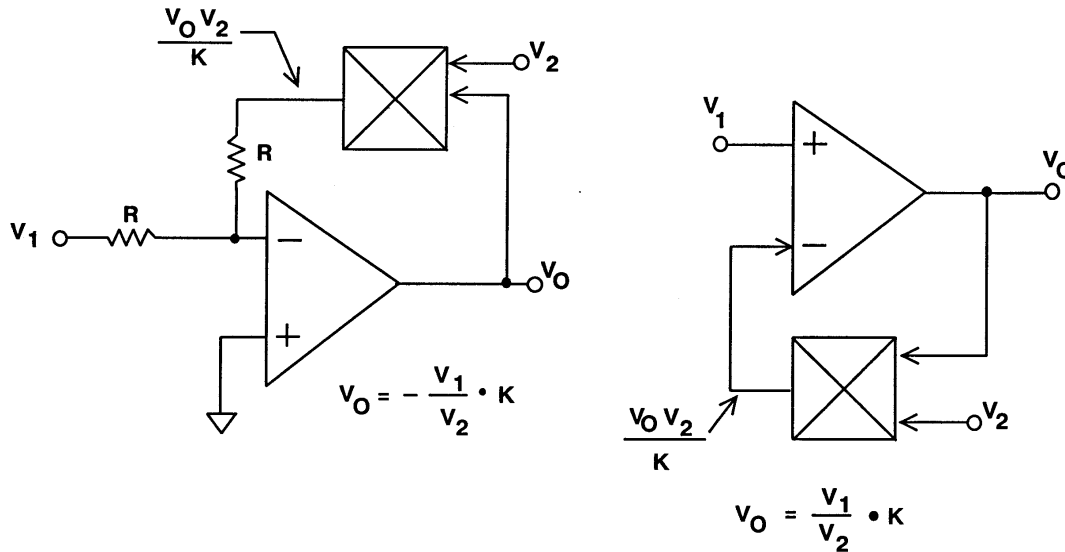


Figure 10: Multipliers used with Op Amps to Perform Division

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2. Barrie Gilbert, *Journal of Solid State Circuits*, Vol. SC-3, December 1968, pp. 353-372.
3. Hank Zumbahlen, *Basic Linear Design*, Analog Devices, 2006, ISBN: 0-915550-28-1. Also available as [Linear Circuit Design Handbook](#), Elsevier-Newnes, 2008, ISBN-10: 0750687037, ISBN-13: 978-0750687034. Chapter 2, 4.

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